# Chapter 13

### *Chapter 13 Maintaining Mathematical Proficiency (p. 703)*

**1.** $$\frac{a}{w} = \frac{p}{100}$$
$$\frac{6}{30} = \frac{p}{100}$$
$$100 \cdot \frac{6}{30} = 100 \cdot \frac{p}{100}$$
$$20 = p$$

So, 6 is 20% of 30.

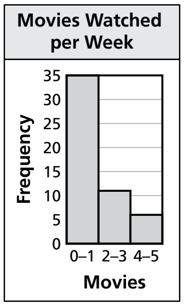
**2.** $$\frac{a}{w} = \frac{p}{100}$$
$$\frac{a}{25} = \frac{68}{100}$$
$$25 \cdot \frac{a}{25} = 25 \cdot \frac{68}{100}$$
$$a = 17$$

So, 17 is 68% of 25.

**3.** $$\frac{a}{w} = \frac{p}{100}$$
$$\frac{34.4}{86} = \frac{p}{100}$$
$$100 \cdot \frac{34.4}{86} = 100 \cdot \frac{p}{100}$$
$$40 = p$$

So, 34.4 is 40% of 86.

**4.** Draw and label the axes. Draw a bar to represent the frequency of each interval.




**5.** They are not equally priced because the sofa will cost 80% (20% off) of the retail price and the arm chair will cost 81% (10% off, then another 10% off) of the retail price.

### *Chapter 13 Mathematical Thinking (p. 704)*

**1.** This is equally likely to happen or not happen. The probability of the event is 0.5.

**2.** This is unlikely. The probability of the event is $(0.5)(0.5) = 0.25$.

**3.** *Sample answer:* Rolling a number less than 7 on a six-sided die is certain to occur.

### *13.1 Explorations (p. 705)*

**1.** The possible outcomes are HHH, HHT, HTH, THH, HTT, THT, TTH, TTT.

**2. a.** The possible outcomes are 1, 2, 3, 4, 5, 6.

**b.** The possible outcomes are 1-1, 1-2, 1-3, 1-4, 1-5, 1-6, 2-1, 2-2, 2-3, 2-4, 2-5, 2-6, 3-1, 3-2, 3-3, 3-4, 3-5, 3-6, 4-1, 4-2, 4-3, 4-4, 4-5, 4-6, 5-1, 5-2, 5-3, 5-4, 5-5, 5-6, 6-1, 6-2, 6-3, 6-4, 6-5, 6-6.

**3. a.** There is 1 way to spin a 1, 2 ways to spin a 2, 3 ways to spin a 3, 2 ways to spin a 4, and 4 ways to spin a 5.

**b.** The sample space is 1, 2, 2, 3, 3, 3, 4, 4, 5, 5, 5, 5.

**c.** There are 12 outcomes.

**4. a.** There are 2 ways to choose two blue marbles, 10 ways to choose a red then blue, 10 ways to choose a blue then red, and 20 ways to choose two red marbles.

**b.** The possible outcomes are B1-B2, B2-B1, B1-R1, B1-R2, B1-R3, B1-R4, B1-R5, B2-R1, B2-R2, B2-R3, B2-R4, B2-R5, R1-B1, R1-B2, R2-B1, R2-B2, R3-B1, R3-B2, R4-B1, R4-B2, R5-B1, R5-B2, R1-R2, R1-R3, R1-R4, R1-R5, R2-R1, R2-R3, R2-R4, R2-R5, R3-R1, R3-R2, R3-R4, R3-R5, R4-R1, R4-R2, R4-R3, R4-R5, R5-R1, R5-R2, R5-R3, R5-R4.

**c.** There are 42 possible outcomes.

**5.** *Sample answer:* You can make a table or diagram to show all of the possible outcomes.

**6.** Exploration 3:

| Outcome | Ratio |
|---|---|
| 1 | $\frac{1}{12}$ |
| 2 | $\frac{2}{12}$ |
| 3 | $\frac{3}{12}$ |
| 4 | $\frac{2}{12}$ |
| 5 | $\frac{4}{12}$ |

The sum is $\frac{1}{12} + \frac{2}{12} + \frac{3}{12} + \frac{2}{12} + \frac{4}{12} = \frac{12}{12} = 1$.

Exploration 4:

| Outcome | Ratio |
|---|---|
| Two blue | $\frac{2}{42}$ |
| Red then blue | $\frac{10}{42}$ |
| Blue then red | $\frac{10}{42}$ |
| Two red | $\frac{20}{42}$ |

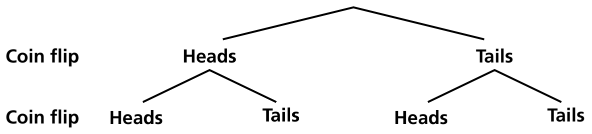
The sum is $\frac{2}{42} + \frac{10}{42} + \frac{10}{42} + \frac{20}{42} = \frac{42}{42} = 1$.

The sum of the ratios for both Explorations is 1.

# Chapter 13

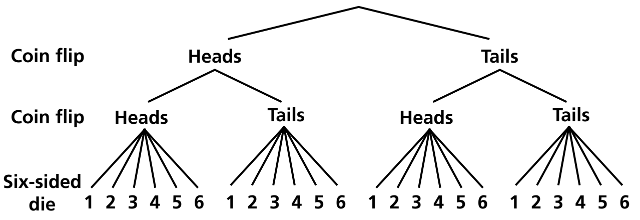
### *13.1 Monitoring Progress (pp. 706–709)*

**1.** Use a tree diagram to find the outcomes in the sample space.



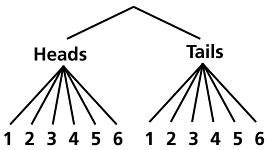

The sample space has 4 possible outcomes. They are HT, HH, TH, and TT.

**2.** Use a tree diagram to find the outcomes in the sample space.




The sample space has 24 possible outcomes. They are HH1, HH2, HH3, HH4, HH5, HH6, HT1, HT2, HT3, HT4, HT5, HT6, TH1, TH2, TH3, TH4, TH5, TH6, TT1, TT2, TT3, TT4, TT5, TT6.

**3.** Find the number of outcomes.




There are 12 outcomes, where one shows tails and 4. So, the probability is $\frac{1}{12}$.

**4.** $P(\overline{A}) = 1 - P(A)$

$= 1 - 0.45$

$= 0.55$

So, $P(\overline{A}) = 0.55$.

**5.** $P(\overline{A}) = 1 - P(A)$

$= 1 - \frac{1}{4}$

$= \frac{3}{4}$

So, $P(\overline{A}) = \frac{3}{4}$.

**6.** $P(\overline{A}) = 1 - P(A)$

$= 1 - 1$

$= 0$

So, $P(\overline{A}) = 0$.

**7.** $P(\overline{A}) = 1 - P(A)$

$= 1 - 0.03$

$= 0.97$

So, $P(A) = 0.97$.

**8.** $P(\text{5 points}) = \dfrac{\text{Area of medium circle} - \text{Area of small circle}}{\text{Area of entire board}}$

$= \dfrac{\pi \cdot 6^2 - \pi \cdot 3^2}{18^2}$

$= \dfrac{27\pi}{324}$

$\approx 0.262$

It is more likely to get 5 points, because the probability is 0.262, while the probability to get 10 points is 0.215.

**9.** $P(\overline{\text{0 points}}) = 1 - P(\text{0 points})$

$\approx 1 - 0.215$

$= 0.785$

It is more likely to score points.

**10.** The experimental probability is greater than the theoretical probability for green.

**11.** $P(\text{pet-owning adult has a fish}) = \dfrac{146}{1328} \approx 0.110$

So, the probability that a pet-owning adult has a fish is about 11%.

### *13.1 Exercises (pp. 710–712)*

#### *Vocabulary and Core Concept Check*

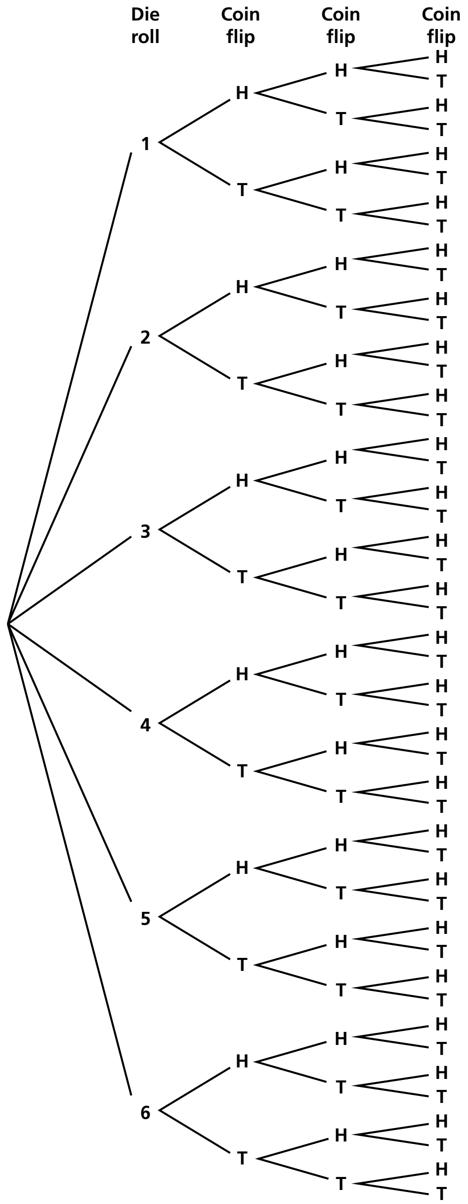
**1.** A number that describes the likelihood of an event is the probability of the event.

**2.** Theoretical probability is based on the number of outcomes and experimental probability is based on repeated trials.

# Chapter 13

***Monitoring Progress and Modeling with Mathematics***

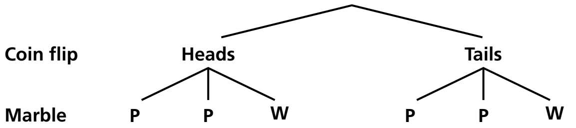
**3.** Use a tree diagram to find the outcomes in the sample space.




The sample space has 48 possible outcomes. They are listed below.

| | | | | | |
|---|---|---|---|---|---|
| 1HHH | 2HHH | 3HHH | 4HHH | 5HHH | 6HHH |
| 1HHT | 2HHT | 3HHT | 4HHT | 5HHT | 6HHT |
| 1HTH | 2HTH | 3HTH | 4HTH | 5HTH | 6HTH |
| 1THH | 2THH | 3THH | 4THH | 5THH | 6THH |
| 1HTT | 2HTT | 3HTT | 4HTT | 5HTT | 6HTT |
| 1THT | 2THT | 3THT | 4THT | 5THT | 6THT |
| 1TTH | 2TTH | 3TTH | 4TTH | 5TTH | 6TTH |
| 1TTT | 2TTT | 3TTT | 4TTT | 5TTT | 6TTT |

**4.** Use a tree diagram to find the outcomes in the sample space.



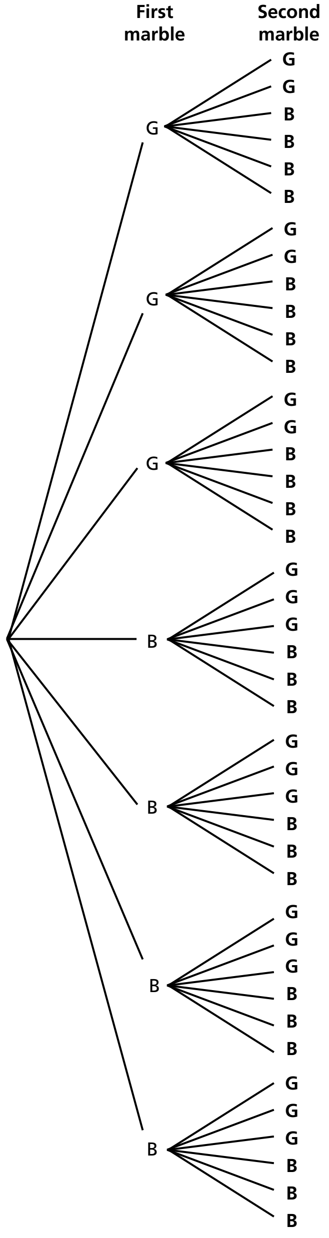

The sample space has 6 possible outcomes. They are listed below.

HP TP HP TP HW TW

**5.** The sample space has 12 possible outcomes. They are listed below.

| | | |
|---|---|---|
| R1 | W1 | B1 |
| R2 | W2 | B2 |
| R3 | W3 | B3 |
| R4 | W4 | B4 |

**6.** Use a tree diagram to find the outcomes in the sample space.




The sample space has 42 possible outcomes. They are:

GG, GG, GG, GG, GG, GG, GB, GB, GB, GB, GB, GB, GB, GB, GB, GB, GB, GB, BG, BG, BG, BG, BG, BG, BG, BG, BG, BG, BG, BG, BB, BB, BB, BB, BB, BB, BB, BB, BB, BB, BB, BB.

**7. Step 1** Find the number of outcomes in the sample space.

Let W represent a win and L represent a lose.

| Number of winners | Outcomes |
|---|---|
| 0 | LLLLL |
| 1 | WLLLL LWLLL LLWLL LLLWL LLLLW |
| 2 | WWLLL WLWLL WLLWL<br>WLLLW LWWLL LWLWL<br>LWLLW LLWWL LLWLW LLLWW |
| 3 | WWWLL WWLWL WWLLW<br>WLWWL WLWLW WLLWW<br>LWWWL LWWLW LWLWW LLWWW |
| 4 | WWWWL WWWLW WWLWW<br>WLWWW LWWWW |
| 5 | WWWWW |

**Step 2** Identify the number of favorable outcomes and the total number of outcomes. There are 10 favorable outcomes and the total number of outcomes is 32.

**Step 3** Find the probability of exactly two of the five contestants winning a prize. So, use the theoretical probability formula.

$$P(\text{exactly two win}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{10}{32} = \frac{5}{16}$$

The probability of exactly two of the five contestants winning is $\frac{5}{16}$, or 31.25%.

**8. Step 1** Find the number of outcomes in the sample space. Let S represent spades, H represent hearts, C represent clubs, and D represent diamonds.

| Number of spades | Outcomes |
|---|---|
| 0 | HD HH HC DH DC DD CH CD CC |
| 1 | SD SH SC |
| 2 | SS |

**Step 2** Identify the number of favorable outcomes and the total number of outcomes. There is 1 favorable outcome and the total number of outcomes is 16.

**Step 3** Find the probability of drawing two spades. So, use the theoretical probability formula.

$$P(\text{two spades}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{16}$$

The probability of drawing two spades is $\frac{1}{16}$, or 6.25%.

**9. a.** $P(\text{sum is not } 4) = 1 - P(\text{sum is } 4)$

$$= 1 - \frac{3}{36}$$
$$= \frac{33}{36}$$
$$= \frac{11}{12}$$
$$\approx 0.92$$

The probability is $\frac{11}{12}$, or about 92%.

**b.** $P(\text{sum is greater than } 5)$

$$= 1 - P(\text{sum less than or equal to } 5)$$
$$= 1 - \frac{10}{36}$$
$$= \frac{26}{36}$$
$$= \frac{13}{18}$$
$$\approx 0.72$$

The probability is $\frac{13}{18}$, or about 72%.

**10. a.** $P(\text{person at least 15 years old})$

$$= 1 - P(\text{person less than 15 years old})$$
$$= 1 - (0.13 + 0.07)$$
$$= 0.80$$

The probability is 0.80, or 80%.

**b.** $P(\text{person whose age is from 25 to 44 years old})$

$$= 0.13 + 0.13$$
$$= 0.26$$

The probability is 0.26, or 26%.

**11.** There are 4 outcomes, not 3. So, the probability is $\frac{1}{4}$.

**12.** The event should be that the number is less than or equal to 4. So, the probability is $\frac{13}{15}$.

**13.** $P(\text{yellow}) = \dfrac{\text{Area of yellow}}{\text{Area of entire board}}$

$$= \frac{\pi \cdot 9^2 - \left(6^2 + 2 \cdot \frac{1}{2}(6)(6)\right)}{18^2}$$
$$= \frac{81\pi - 72}{324}$$
$$\approx 0.56$$

So, the probability is 0.56, or about 56%.

**14. a.** $P(\text{Texas}) = \dfrac{\text{Length of Texas shoreline}}{\text{Length of shoreline}}$

$$= \frac{367}{1631}$$
$$\approx 0.23$$

So, the probability is about 0.23, or about 23%.

**b.** $P(\text{Alabama}) = \dfrac{\text{Length of Alabama shoreline}}{\text{Length of shoreline}}$

$$= \frac{53}{1631}$$
$$\approx 0.03$$

So, the probability is about 0.03, or about 3%.

**c.** $P(\text{Florida}) = \dfrac{\text{Length of Florida shoreline}}{\text{Length of shoreline}}$

$$= \frac{770}{1631}$$
$$\approx 0.47$$

So, the probability is about 0.47, or about 47%.

**d.** $P(\text{Louisiana}) = \dfrac{\text{Length of Louisiana shoreline}}{\text{Length of shoreline}}$

$$= \frac{397}{1631}$$
$$\approx 0.24$$

So, the probability is about 0.24, or about 24%.

**15.** The theoretical probability of rolling each number is $\frac{1}{6}$.

Use the outcomes in the table to find the experimental probabilities.

$P(1) = \frac{11}{60}$ $\quad P(3) = \frac{7}{60}$ $\quad P(5) = \frac{6}{60}$

$P(2) = \frac{14}{60}$ $\quad P(4) = \frac{10}{60}$ $\quad P(6) = \frac{12}{60}$

Four has the same experimental probability as the theoretical probability.

**16.** The theoretical probability of drawing each marble is $\frac{1}{5}$. Use the outcomes in the table to find the experimental probability.

$P(\text{white}) = \frac{5}{30}$ $\quad P(\text{green}) = \frac{2}{30}$ $\quad P(\text{black}) = \frac{6}{30}$

$P(\text{blue}) = \frac{9}{30}$ $\quad P(\text{red}) = \frac{8}{30}$

The black marble has the same experimental probability as the theoretical probability.

**17. a.** $P(\text{multiple of } 3) = \frac{9}{10}$

So, the probability is $\frac{9}{10}$, or 90%.

**b.** $P(\text{multiple of } 3) = \frac{20}{30} = \frac{2}{3}$

So, the probability is $\frac{2}{3}$, or about 67%.

**c.** The probability in part (b) is based on trials, not possible outcomes.

**18.** *Sample answer:* Drawing an orange marble from a bag containing red marbles has a probability of 0 and drawing a red marble from a bag containing red marbles has a probability of 1.

**19.** $P(\text{favorite sport is auto racing}) = \frac{179}{2237} \approx 0.08$

So, the probability is about 0.08, or about 8%.

**20.** $P(\text{like Italian}) = \frac{526}{2392} \approx 0.22$

So, the probability is about 0.22, or about 22%.

**21.** $$P(\text{green}) = \frac{\text{Area of green}}{\text{Area of entire board}} = \frac{2 \cdot \frac{1}{2} \cdot 6 \cdot 6}{18^2} = \frac{36}{324} \approx 0.11$$

$$P(\text{not blue}) = \frac{\text{Area of large square} - \text{Area of large circle}}{\text{Area of entire board}} = \frac{18^2 - \pi \cdot 9^2}{18^2} = \frac{324 - 81\pi}{324} \approx 0.83$$

$$P(\text{red}) = \frac{\text{Area of red}}{\text{Area of entire board}} = \frac{\pi \cdot 3^2}{18^2} = \frac{9\pi}{324} \approx 0.09$$

From Exercise 13, $P(\text{yellow}) \approx 0.56$.
So, $P(\text{not yellow}) \approx 1 - 0.56 = 0.44$.

So, the order is C, A, D, and B.

**22.** $P(\text{rain on Sunday}) = 80\%$

$P(\text{no rain on Saturday}) = 1 - P(\text{rain on Saturday})$
$= 1 - 0.3 = 0.7$, or 70%

$P(\text{rain on Monday}) = 90\%$

$P(\text{no rain on Friday}) = 1 - P(\text{rain on Friday})$
$= 1 - 0.05 = 0.95$, or 95%

So, the order is B, A, C, and D.

**23. a.** The possible sums are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12.

**b.**

| Sum | Probability |
|---|---|
| 2 | $\frac{1}{36}$ |
| 3 | $\frac{2}{36} = \frac{1}{18}$ |
| 4 | $\frac{3}{36} = \frac{1}{12}$ |
| 5 | $\frac{4}{36} = \frac{1}{9}$ |
| 6 | $\frac{5}{36}$ |
| 7 | $\frac{6}{36} = \frac{1}{6}$ |
| 8 | $\frac{5}{36}$ |
| 9 | $\frac{4}{36} = \frac{1}{9}$ |
| 10 | $\frac{3}{36} = \frac{1}{12}$ |
| 11 | $\frac{2}{36} = \frac{1}{18}$ |
| 12 | $\frac{1}{36}$ |

**c.** *Sample answer:* The probabilities are similar.

**24.** Your friend is incorrect. Your friend calculated the experimental probability. The theoretical probability of the coin landing heads up is $\frac{1}{2}$.

**25.** $$P(\text{point is sphere}) = \frac{\text{Volume of sphere}}{\text{Volume of cube}} = \frac{\frac{4}{3}\pi \cdot \left(\frac{1}{2}\right)^3}{1^2} = \frac{\pi}{6}$$

So, the probability is $\frac{\pi}{6}$, or about 52%.

**26.** The probability is $\frac{2}{3}$ since $f(x) + c$ intersects the $x$-axis when $c$ is 1, 2, 3, or 4.

**27.** $$P(\text{defective computer}) = \frac{9}{1200} = \frac{3}{400}$$

So, the probability is $\frac{3}{400}$, or 0.75%. When 15,000 computers are shipped, $(0.0075)(15{,}000) = 112.5$, or about 113 are defective.

**28.** *Sample answer:* Box A contains three cards numbered 1, 2, and 3, and Box B contains two cards numbered 1 and 2. One card is removed at random from each box. Find the probability that the product of two numbers is at least 5. The answer is $\frac{1}{6}$.

### *Maintaining Mathematical Proficiency*

**29.** $-\frac{\cancel{3}^{1}}{{}_{4}\cancel{8}} \cdot \left(-\frac{\cancel{2}^{1}}{\cancel{9}_{3}}\right) = \frac{1}{12}$

**30.** $\frac{7}{10} \cdot \left(-1\frac{4}{11}\right) \cdot \frac{1}{3} = \frac{7}{{}_{2}\cancel{10}} \cdot \left(-\frac{\cancel{\cancel{15}}^{3}}{11}\right) \cdot \frac{1}{\cancel{3}_{1}} = -\frac{7}{22}$

**31.** $1\frac{5}{6} \div \frac{7}{12} \cdot \frac{4}{5} = \frac{11}{{}_{1}\cancel{6}} \cdot \frac{\cancel{12}^{2}}{7} \cdot \frac{4}{5}$

$= \frac{88}{35}$, or $2\frac{18}{35}$

**32.** $-3.2 + 7.8 \cdot (-2.2) = -3.2 + (-17.16)$

$= -20.36$

**33.** $-\frac{1}{6}\left(2\frac{1}{4}\right) - \frac{1}{2} = -\frac{1}{{}_{2}\cancel{6}}\left(\frac{\cancel{9}^{3}}{4}\right) - \frac{1}{2}$

$= -\frac{3}{8} - \frac{1}{2}$

$= -\frac{3}{8} - \frac{4}{8}$

$= -\frac{7}{8}$

**34.** $4.9 - 5.1 \div 0.2^2 = 4.9 - 5.1 \div 0.04$

$= 4.9 - 127.5$

$= -122.6$

### *13.2 Explorations (p. 713)*

**1. a.** They are independent because the occurrence of one event does not affect the occurrence of the other event.

**b.** They are dependent because the occurrence of one event does affect the occurrence of the other event.

**2. a.** *Sample answer:* The probability is 0.2. This occurs when two six-sided dice are rolled, and the sum of the numbers are recorded.

**b.** *Sample answer:* The probability is 0.2. This occurs when two pieces of paper are selected one at a time numbered 1 through 6, and the sum of the numbers are recorded.

**3. a.** The probability is $\frac{6}{36} = \frac{1}{6} \approx 0.167$. *Sample answer:* This is less than the probability in Exploration 2(a).

**b.** The probability is $\frac{6}{30} = \frac{1}{5} = 0.2$. *Sample answer:* This is the same as the probability in Exploration 2(b).

**c.** The probability in part (a) is less than the probability in part (b).

**4.** Determine whether the occurrence of one event does not affect the occurrence of the other event.

**5. a.** They are independent because the occurrence of one event does not affect the occurrence of the other event.

**b.** They are dependent because the occurrence of one event does affect the occurrence of the other event.

### *13.2 Monitoring Progress (pp. 715–717)*

**1.** The events are independent.

**2.** The events are dependent.

**3.** $P(\text{even and odd}) = P(\text{even}) \cdot P(\text{odd})$

$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

The probability is $\frac{1}{4}$, or 25%.

**4.** The events are dependent because there is one less bill in the bag on your second draw than on your first draw. Find $P(\$1)$ and $P(\$1 \mid \$1)$. Then multiply the probabilities.

$P(\$1) = \frac{20}{25}$

$P(\$1 \mid \$1) = \frac{19}{24}$

$P(\$1 \text{ and } \$1) = P(\$1) \cdot P(\$1 \mid \$1)$

$= \frac{20}{25} \cdot \frac{19}{24}$

$= \frac{19}{30}$

$\approx 0.633$

So, the probability that you draw two \$1 bills is about 63.3%.

**5. a.** Because you replace each card before you select the next card, the events are independent. So, the probability is $P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C)$

$= \frac{39}{52} \cdot \frac{39}{52} \cdot \frac{39}{52}$

$= \frac{27}{64}$

$\approx 0.422$, or about 42.2%.

**b.** Because you do not replace each card before you select the next card, the events are dependent. So, the probability is $P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B \mid A) \cdot P(C \mid A \text{ and } C)$

$= \frac{39}{52} \cdot \frac{38}{51} \cdot \frac{37}{50}$

$= \frac{703}{1700}$

$\approx 0.414$, or about 41.4%.

You are about $\frac{27}{64} \div \frac{703}{1700} \approx 1.02$ times more likely to select 3 cards that are not hearts when you replace each card before you select the next card.