Chapter 13

28. *Sample answer:* Box A contains three cards numbered 1, 2, and 3, and Box B contains two cards numbered 1 and 2. One card is removed at random from each box. Find the probability that the product of two numbers is at least 5. The answer is $\frac{1}{6}$.

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29. $-\frac{{}^{1}\cancel{3}}{{}_{4}\cancel{8}} \cdot \left(-\frac{\cancel{2}^{1}}{\cancel{9}_{3}}\right) = \frac{1}{12}$

30. $\frac{7}{10} \cdot \left(-1\frac{4}{11}\right) \cdot \frac{1}{3} = \frac{7}{{}_{2}\cancel{10}} \cdot \left(-\frac{\cancel{15}^{3}}{11}\right) \cdot \frac{1}{\cancel{3}_{1}} = -\frac{7}{22}$

31. $1\frac{5}{6} \div \frac{7}{12} \cdot \frac{4}{5} = \frac{11}{{}_{1}\cancel{6}} \cdot \frac{\cancel{12}^{2}}{7} \cdot \frac{4}{5}$

$= \frac{88}{35}$, or $2\frac{18}{35}$

32. $-3.2 + 7.8 \cdot (-2.2) = -3.2 + (-17.16)$

$= -20.36$

33. $-\frac{1}{6}\left(2\frac{1}{4}\right) - \frac{1}{2} = -\frac{1}{{}_{2}\cancel{6}}\left(\frac{\cancel{9}^{3}}{4}\right) - \frac{1}{2}$

$= -\frac{3}{8} - \frac{1}{2}$

$= -\frac{3}{8} - \frac{4}{8}$

$= -\frac{7}{8}$

34. $4.9 - 5.1 \div 0.2^2 = 4.9 - 5.1 \div 0.04$

$= 4.9 - 127.5$

$= -122.6$

13.2 Explorations (p. 713)

1. a. They are independent because the occurrence of one event does not affect the occurrence of the other event.

b. They are dependent because the occurrence of one event does affect the occurrence of the other event.

2. a. *Sample answer:* The probability is 0.2. This occurs when two six-sided dice are rolled, and the sum of the numbers are recorded.

b. *Sample answer:* The probability is 0.2. This occurs when two pieces of paper are selected one at a time numbered 1 through 6, and the sum of the numbers are recorded.

3. a. The probability is $\frac{6}{36} = \frac{1}{6} \approx 0.167$. *Sample answer:* This is less than the probability in Exploration 2(a).

b. The probability is $\frac{6}{30} = \frac{1}{5} = 0.2$. *Sample answer:* This is the same as the probability in Exploration 2(b).

c. The probability in part (a) is less than the probability in part (b).

4. Determine whether the occurrence of one event does not affect the occurrence of the other event.

5. a. They are independent because the occurrence of one event does not affect the occurrence of the other event.

b. They are dependent because the occurrence of one event does affect the occurrence of the other event.

13.2 Monitoring Progress (pp. 715–717)

1. The events are independent.

2. The events are dependent.

3. $P(\text{even and odd}) = P(\text{even}) \cdot P(\text{odd})$

$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

The probability is $\frac{1}{4}$, or 25%.

4. The events are dependent because there is one less bill in the bag on your second draw than on your first draw. Find $P(\$1)$ and $P(\$1 \mid \$1)$. Then multiply the probabilities.

$P(\$1) = \frac{20}{25}$

$P(\$1 \mid \$1) = \frac{19}{24}$

$P(\$1 \text{ and } \$1) = P(\$1) \cdot P(\$1 \mid \$1)$

$= \frac{20}{25} \cdot \frac{19}{24}$

$= \frac{19}{30}$

≈ 0.633

So, the probability that you draw two \$1 bills is about 63.3%.

5. a. Because you replace each card before you select the next card, the events are independent. So, the probability is $P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C)$

$= \frac{39}{52} \cdot \frac{39}{52} \cdot \frac{39}{52}$

$= \frac{27}{64}$

≈ 0.422, or about 42.2%.

b. Because you do not replace each card before you select the next card, the events are dependent. So, the probability is $P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B \mid A) \cdot P(C \mid A \text{ and } C)$

$= \frac{39}{52} \cdot \frac{38}{51} \cdot \frac{37}{50}$

$= \frac{703}{1700}$

≈ 0.414, or about 41.4%.

You are about $\frac{27}{64} \div \frac{703}{1700} \approx 1.02$ times more likely to select 3 cards that are not hearts when you replace each card before you select the next card.

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6. a. $P(\text{pass} \mid \text{non-defective})$

$$= \frac{\text{Number of non-defective parts "passed"}}{\text{Total number of non-defective}}$$

$$= \frac{450}{450 + 11} = \frac{450}{461} \approx 0.976, \text{ or about } 97.6\%$$

b. $P(\text{fails} \mid \text{defective})$

$$= \frac{\text{Number of defective parts "fails"}}{\text{Total number of defective}}$$

$$= \frac{36}{3 + 36} = \frac{36}{39} \approx 0.923, \text{ or about } 92.3\%$$

7. Let event A be "order coffee" and let event B be "order coffee and a bagel." You are given $P(A) = 0.8$ and $P(A \text{ and } B) = 0.15$. Use the formula to find $P(B \mid A)$.

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

$$= \frac{0.15}{0.8}$$

$$= \frac{3}{16}$$

$$\approx 0.1875$$

So, the probability that a customer who orders coffee also orders a bagel is 18.75%.

13.2 Exercises (pp. 718–720)

Vocabulary and Core Concept Check

1. When two events are dependent, the occurrence of one event affects the other. When two events are independent, the occurrence of one event does not affect the other. *Sample answer:* choosing two marbles from a bag without replacement; rolling two dice

2. The probability that event B will occur given that event A has occurred is called the conditional probability of B given A and is written as $P(B \mid A)$.

Monitoring Progress and Modeling with Mathematics

3. The events are dependent because the occurrence of event A affects the occurrence of event B.

4. The events are independent because the occurrence of event A does not affect the occurrence of event B.

5. The events are dependent because the occurrence of event A affects the occurrence of event B.

6. The events are independent because the occurrence of event A does not affect the occurrence of event B.

7. Let Y represent yellow, G represent green, R represent red, and B represent blue. Use a table to list the outcomes in the sample space.

Y Y	G Y	R Y	B Y
Y G	G G	R G	B G
Y R	G R	R R	B R
Y B	G B	R B	B B

$P(\text{blue}) = \frac{1}{4}$ and $P(\text{green}) = \frac{1}{4}$

$P(\text{blue and then green}) = \frac{1}{16}$

Because $\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$, the events are independent.

8. Let R represent the red apple and G_1, G_2, and G_3 represent the green apples. Use a table to list the outcomes in the sample space.

R G_1	G_1 R	G_2 R	G_3 R
R G_2	G_1 G_2	G_2 G_1	G_3 G_1
R G_3	G_1 G_3	G_2 G_3	G_3 G_2

$P(\text{red}) = \frac{1}{4}$ and $P(\text{green}) = \frac{3}{4}$

$P(\text{red and then green}) = \frac{3}{12} = \frac{1}{4}$

Because $\frac{1}{4} \cdot \frac{3}{4} \neq \frac{1}{4}$, the events are not independent.

9. Let C represent correct and I represent incorrect. Use a table to list the outcomes in the sample space.

I I	C I
I C	C C

$P(\text{question 1}) = \frac{1}{2}$ and $P(\text{question 2}) = \frac{1}{2}$

$P(\text{question 1 and question 2}) = \frac{1}{4}$

Because $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$, the events are independent.

10. Let R represent the red rose and W_1, W_2, W_3, and W_4 represent the white roses. Use a table to list the outcomes in the sample space.

RW_1	W_1R	W_2R	W_3R	W_4R
RW_2	W_1W_2	W_2W_1	W_3W_1	W_4W_1
RW_3	W_1W_3	W_2W_3	W_3W_2	W_4W_2
RW_4	W_1W_4	W_2W_4	W_3W_4	W_4W_3

$P(\text{red}) = \frac{1}{5}$ and $P(\text{white}) = \frac{4}{5}$

$P(\text{white and white}) = \frac{12}{20} = \frac{1}{10}$

Because $\frac{1}{5} \cdot \frac{4}{5} \neq \frac{1}{10}$, the events are not independent.

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11. Let event A be "more than \$500" and let B be "bankrupt." The events are independent because the outcome of your second spin is not affected by the outcome of your first spin. Find the probability of each event and then multiply the probabilities.

$P(A) = \frac{8}{24}$

$P(B) = \frac{2}{24}$

$P(A \text{ and } B) = P(A) \cdot P(B) = \frac{8}{24} \cdot \frac{2}{24} = \frac{1}{36} \approx 0.028$

So, the probability that you spin more than \$500 on your first spin and bankrupt on your second spin is about 2.8%.

12. Let event A be "draw 3" and let B be "draw greater than 10." The events are independent because the outcome of your second draw is not affected by the outcome of your first draw. Find the probability of each event and then multiply the probabilities.

$P(A) = \frac{1}{25}$

$P(B) = \frac{15}{25}$

$P(A \text{ and } B) = P(A) \cdot P(B) = \frac{1}{25} \cdot \frac{15}{25} = \frac{3}{125} = 0.024$

So, the probability that you draw 3 with your first draw and a number greater than 10 with your second draw is 2.4%.

13. The events are dependent because there is one less sock in the drawer on your second draw. Find $P(A)$ and $P(B|A)$. Then multiply the probabilities.

$P(A) = \frac{12}{20}$

$P(B|A) = \frac{11}{19}$

$P(A \text{ and } B) = P(A) \cdot P(B|A) = \frac{12}{20} \cdot \frac{11}{19} = \frac{33}{95} \approx 0.347$

So, the probability that you draw two white socks is about 34.7%.

14. The events are dependent because there is one less tile on your second draw than on your first draw. Find $P(A)$ and $P(B|A)$. Then multiply the probabilities.

$P(A) = \frac{56}{100}$

$P(B|A) = \frac{42}{100}$

$P(A \text{ and } B) = P(A) \cdot P(B|A) = \frac{56}{100} \cdot \frac{42}{100} = \frac{147}{625} \approx 0.238$

So, the probability that you draw a consonant and vowel is about 23.8%.

15. The probabilities were added instead of multiplied.

$P(A \text{ and } B) = (0.6)(0.2) = 0.12$

16. $P(B|A)$ is incorrect.

$P(B|A) = \frac{4}{6}$

$P(A \text{ and } B) = \frac{2}{7} \approx 0.286$

17. $P(A \text{ and } B) = P(A) \cdot P(B)$

$0.13 = P(A)(0.4)$

$0.325 = P(A)$

So, $P(A) = 0.325$.

18. $P(A \text{ and } B) = P(A) \cdot P(B|A)$

$0.15 = P(A)(0.6)$

$0.25 = P(A)$

So, $P(A) = 0.25$.

19. Let event A be "first face card," event B be "second face card," and event C be "third face card."

a. Because you replace each card before you select the next card, the events are independent. So, the probability is

$$\begin{aligned} P(A \text{ and } B \text{ and } C) &= P(A) \cdot P(B) \cdot P(C) \\ &= \frac{12}{52} \cdot \frac{12}{52} \cdot \frac{12}{52} \\ &= \frac{27}{2197} \approx 0.012, \text{ or about } 1.2\%. \end{aligned}$$

b. Because you do not replace each card before you select the next card, the events are dependent. So, the probability is

$$\begin{aligned} P(A \text{ and } B \text{ and } C) &= P(A) \cdot P(B|A) \cdot P(C|A \text{ and } B) \\ &= \frac{12}{52} \cdot \frac{11}{51} \cdot \frac{10}{50} = \frac{11}{1105} \\ &\approx 0.01, \text{ or about } 1.0\%. \end{aligned}$$

So, you are about $\frac{27}{2197} \cdot \frac{11}{1105} \approx 1.2$ times more likely to select 3 face cards when you replace each card before you select the next card.

20. Let event A be "red marble."

a. Because you replace each marble before you select the next marble, the events are independent. So, the probability is

$$\begin{aligned} P(A \text{ and } A \text{ and } A) &= P(A) \cdot P(A) \cdot P(A) \\ &= \frac{9}{20} \cdot \frac{9}{20} \cdot \frac{9}{20} = \frac{724}{8000} \\ &\approx 0.091, \text{ or about } 9.1\%. \end{aligned}$$

b. Because you do not replace each marble before you select the next marble, the events are dependent. So, the probability is

$$P(A \text{ and } A \text{ and } A) = P(A) \cdot P(A|A) \cdot P(A|A \text{ and } A)$$
$$= \frac{9}{20} \cdot \frac{8}{19} \cdot \frac{7}{18} = \frac{7}{95}$$
$$\approx 0.074, \text{ or about } 7.4\%.$$

You are about $\frac{724}{8000} \cdot \frac{7}{95} \approx 1.23$ times more likely to select 3 red marbles when you replace each marble before you select the next marble.

21. a. $P(\text{bird}|\text{endangered}) = \frac{\text{Number of endangered birds}}{\text{Number of endangered}}$
$$= \frac{80}{70 + 80 + 318}$$
$$= \frac{80}{468}$$
$$\approx 0.171, \text{ or about } 17.1\%$$

b. $P(\text{endangered}|\text{mammals})$
$$= \frac{\text{Number of endangered mammals}}{\text{Number of mammals}}$$
$$= \frac{70}{70 + 16}$$
$$= \frac{70}{86}$$
$$\approx 0.814, \text{ or about } 81.4\%$$

22. a. $P(\text{hurricane}|\text{Northern Hemisphere})$
$$= \frac{\text{Number of Northern Hemisphere hurricanes}}{\text{Number of Northern Hemisphere cyclones}}$$
$$= \frac{379}{100 + 342 + 379}$$
$$= \frac{379}{821}$$
$$\approx 0.462, \text{ or about } 46.2\%$$

b. $P(\text{Southern Hemisphere}|\text{hurricane})$
$$= \frac{\text{Number of Southern Hemisphere hurricanes}}{\text{Number of hurricanes}}$$
$$= \frac{525}{379 + 525}$$
$$= \frac{525}{904}$$
$$\approx 0.581, \text{ or about } 58.1\%$$

23. Let event A be "attended homecoming" and let event B be "attended game and homecoming." You are given $P(A) = 0.43$ and $P(A \text{ and } B) = 0.23$. Use the formula to find $P(B|A)$.

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$
$$= \frac{0.43}{0.23} \approx 0.535$$

So, the probability that a student who attended homecoming also attended the game is about 53.5%.

24. Let event A be "buy gasoline" and let event B be "buy gasoline and a beverage." You are given $P(A) = 0.84$ and $P(B|A) = 0.05$.

Use the formula to find $P(B|A)$.

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$
$$= \frac{0.05}{0.84} \approx 0.060$$

So, the probability that a customer who bought gasoline also bought a beverage is about 6.0%.

25. a. *Sample answer:* Put 20 pieces of paper with each of the 20 students' names in a hat and pick one. The probability is 5%.

b. *Sample answer:* Put 45 pieces of paper in a hat with each student's name appearing once for each hour the student worked. Pick one piece. The probability is about 8.9%.

26. a. Because the second draw is a different color from the first draw, this is without replacement.

b. Because the second draw may contain the same color as the first draw, this is with replacement.

27. Your friend is correct. The chance that the game will be rescheduled is $(0.7)(0.75) = 0.525$, which is greater than a 50% chance.

28. Event A represents rolling at least one 2 and event B represents the dice summing to 5. The events are dependent since $P(A \text{ and } B) = \frac{2}{36}$ and $P(A)P(B) = \frac{11}{324}$.

29. a. The probability that the team wins is 0%, that the team loses is $(0.01)(0.99) = 0.0199 = 1.99\%$, and that the team ties is $(0.99)(0.99) = 0.9801 = 98.01\%$.

b. The probability that the team wins is $(0.45)(0.45) = 0.2025 = 20.25\%$, that the team loses is $(0.55)(0.55) = 0.3025 = 30.25\%$, and that the team ties is 49.5%.

c. The strategy is go for 2 points after the first touchdown, and go for 1 point if they were successful the first time or 2 points if they were unsuccessful the first time. The probability of winning is 44.55% and losing is 30.25%.

30. a. The occurrence of one event does not affect the occurrence of the other, so the probability of each event is the same whether the event has occurred.

b. Yes, it can be $P(A \text{ and } B) = P(A) \bullet P(B)$ and $P(A) = P(A \mid B)$.

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31. $\frac{9}{10}x = 0.18$ Check: $\frac{9}{10}(0.2) \stackrel{?}{=} 0.18$

$9x = 1.8$ $0.18 = 0.18$ ✓

$x = 0.2$

The solution is $x = 0.2$.

32. $\frac{1}{4}x + 0.5x = 1.5$ Check: $\frac{1}{4}(2) + 0.5(2\ ) \stackrel{?}{=} 1.5$

$6.75x = 1.5$ $\frac{1}{2} + 1 \stackrel{?}{=} 1.5$

$x = 2$ $1.5 = 1.5$ ✓

The solution is $x = 2$.

33. $0.3x - \frac{3}{5}x + 1.6 = 1.555$

$-0.3x = -0.045$

$x = 0.15$

Check: $0.3(0.15) - \frac{3}{5}(0.15) + 1.6 = 1.555$

$0.045 - 0.09 + 1.6 \stackrel{?}{=} 1.555$

$1.555 = 1.555$ ✓

The solution is $x = 0.15$.

13.3 Explorations (p. 721)

1.

	Play an Instrument	Do Not Play an Instrument	Total
Speak a Foreign Language	16	30	46
Do Not Speak a Foreign Language	25	9	34
Total	41	39	80

a. 41 students play an instrument.

b. 46 students speak a foreign language.

c. 16 students play an instrument and speak a foreign language.

d. 9 students do not play an instrument and do not speak a foreign language.

e. 25 students play an instrument and do not speak a foreign language.

2. a. $P(\text{instrument}) = \frac{41}{80}$

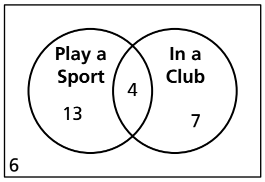
b. $P(\text{foreign language}) = \frac{46}{80} = \frac{23}{40}$

c. $P(\text{instrument and foreign language}) = \frac{16}{80} = \frac{1}{5}$

d. $P(\text{not instrument and not foreign language}) = \frac{9}{80}$

e. $P(\text{instrument and not foreign language}) = \frac{25}{80} = \frac{5}{16}$

3. *Sample answer:*



	Play a Sport	Do Not Play a Sport	Total
In a Club	4	7	11
Not in a Club	13	6	19
Total	17	13	30

Of the students in the class:

43.33% only play a sport

13.33% play a sport and are in a club

23.33% are only in a club

20% do neither activity

a total of 36.67% are in a club

a total of 56.67% play a sport

4. *Sample answer*: You can use a Venn diagram to construct the two-way table. Each entry represents the number of people in each category.

5. You can divide each number in each category by the total surveyed.

13.3 Monitoring Progress (pp. 722–725)

1. Step 1 Find the joint frequencies. Because 61 of the 96 boys are in favor, $96 - 61 = 35$ boys are against. Because 17 of the 88 girls are against, $88 - 17 = 71$ girls are in favor. Place each joint frequency in its corresponding cell.

Step 2 Find the marginal frequencies. Create a new column and row for the sums. Then add the entries and interpret the results.

		Response		
		In Favor	**Against**	**Total**
Gender	**Boy**	61	35	96
	Girl	71	17	88
	Total	132	52	184

184 students were surveyed, 132 students are in favor, 52 students are against, 96 boys were surveyed, 88 girls were surveyed.

Step 3 Find the sums of the marginal frequencies. Notice the sums $96 + 88 = 184$ and $132 + 52 = 184$ are equal. Place this value at the bottom right.