# 9.1 The Pythagorean Theorem

**Essential Question** How can you prove the Pythagorean Theorem?

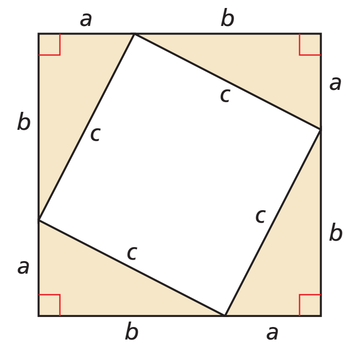
**Texas Essential Knowledge and Skills**
G.6.D
G.9.B

## EXPLORATION 1 Proving the Pythagorean Theorem without Words

**Work with a partner.**

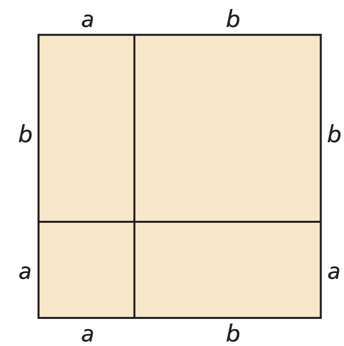
**a.** Draw and cut out a right triangle with legs $a$ and $b$, and hypotenuse $c$.

**b.** Make three copies of your right triangle. Arrange all four triangles to form a large square, as shown.




**c.** Find the area of the large square in terms of $a$, $b$, and $c$ by summing the areas of the triangles and the small square.

**d.** Copy the large square. Divide it into two smaller squares and two equally-sized rectangles, as shown.



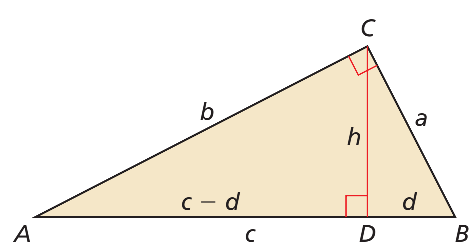

**e.** Find the area of the large square in terms of $a$ and $b$ by summing the areas of the rectangles and the smaller squares.

**f.** Compare your answers to parts (c) and (e). Explain how this proves the Pythagorean Theorem.

## EXPLORATION 2 Proving the Pythagorean Theorem

**Work with a partner.**

**a.** Draw a right triangle with legs $a$ and $b$, and hypotenuse $c$, as shown. Draw the altitude from $C$ to $\overline{AB}$. Label the lengths, as shown.




**REASONING**
To be proficient in math, you need to know and flexibly use different properties of operations and objects.

**b.** Explain why $\triangle ABC$, $\triangle ACD$, and $\triangle CBD$ are similar.

**c.** Write a two-column proof using the similar triangles in part (b) to prove that $a^2 + b^2 = c^2$.

## Communicate Your Answer

**3.** How can you prove the Pythagorean Theorem?

**4.** Use the Internet or some other resource to find a way to prove the Pythagorean Theorem that is different from Explorations 1 and 2.

# 9.1 Lesson

## Core Vocabulary

Pythagorean triple, *p. 468*

***Previous***
right triangle
legs of a right triangle
hypotenuse

## What You Will Learn

- Use the Pythagorean Theorem.
- Use the Converse of the Pythagorean Theorem.
- Classify triangles.

## Using the Pythagorean Theorem

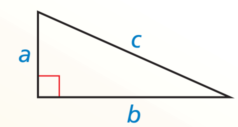
One of the most famous theorems in mathematics is the Pythagorean Theorem, named for the ancient Greek mathematician Pythagoras. This theorem describes the relationship between the side lengths of a right triangle.

### Theorem

**Theorem 9.1 Pythagorean Theorem**

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

*Proof* Explorations 1 and 2, p. 467; Ex. 39, p. 488




$c^2 = a^2 + b^2$

A **Pythagorean triple** is a set of three positive integers $a$, $b$, and $c$ that satisfy the equation $c^2 = a^2 + b^2$.

### STUDY TIP

You may find it helpful to memorize the basic Pythagorean triples, shown in **bold**, for standardized tests.

### Core Concept

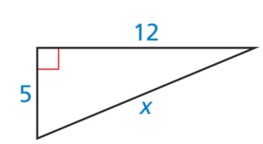
**Common Pythagorean Triples and Some of Their Multiples**

| **3, 4, 5** | **5, 12, 13** | **8, 15, 17** | **7, 24, 25** |
|---|---|---|---|
| 6, 8, 10 | 10, 24, 26 | 16, 30, 34 | 14, 48, 50 |
| 9, 12, 15 | 15, 36, 39 | 24, 45, 51 | 21, 72, 75 |
| $3x$, $4x$, $5x$ | $5x$, $12x$, $13x$ | $8x$, $15x$, $17x$ | $7x$, $24x$, $25x$ |

The most common Pythagorean triples are in bold. The other triples are the result of multiplying each integer in a bold-faced triple by the same factor.

### EXAMPLE 1 Using the Pythagorean Theorem

Find the value of $x$. Then tell whether the side lengths form a Pythagorean triple.



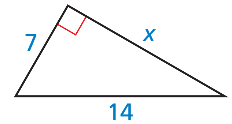

**SOLUTION**

| | |
|---|---|
| $c^2 = a^2 + b^2$ | Pythagorean Theorem |
| $x^2 = 5^2 + 12^2$ | Substitute. |
| $x^2 = 25 + 144$ | Multiply. |
| $x^2 = 169$ | Add. |
| $x = 13$ | Find the positive square root. |

- The value of $x$ is 13. Because the side lengths 5, 12, and 13 are integers that satisfy the equation $c^2 = a^2 + b^2$, they form a Pythagorean triple.

## EXAMPLE 2 Using the Pythagorean Theorem

Find the value of $x$. Then tell whether the side lengths form a Pythagorean triple.




### SOLUTION

$$c^2 = a^2 + b^2$$ Pythagorean Theorem

$$14^2 = 7^2 + x^2$$ Substitute.

$$196 = 49 + x^2$$ Multiply.

$$147 = x^2$$ Subtract 49 from each side.

$$\sqrt{147} = x$$ Find the positive square root.

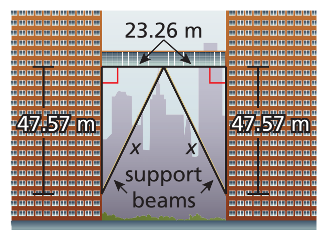
$$\sqrt{49} \bullet \sqrt{3} = x$$ Product Property of Radicals

$$7\sqrt{3} = x$$ Simplify.

▶ The value of $x$ is $7\sqrt{3}$. Because $7\sqrt{3}$ is not an integer, the side lengths do not form a Pythagorean triple.

## EXAMPLE 3 Solving a Real-Life Problem

The skyscrapers shown are connected by a skywalk with support beams. Use the Pythagorean Theorem to approximate the length of each support beam.




### SOLUTION

Each support beam forms the hypotenuse of a right triangle. The right triangles are congruent, so the support beams are the same length.

$$x^2 = (23.26)^2 + (47.57)^2$$ Pythagorean Theorem

$$x = \sqrt{(23.26)^2 + (47.57)^2}$$ Find the positive square root.

$$x \approx 52.95$$ Use a calculator to approximate.

▶ The length of each support beam is about 52.95 meters.

## Monitoring Progress

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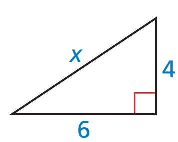
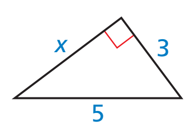
**Find the value of $x$. Then tell whether the side lengths form a Pythagorean triple.**

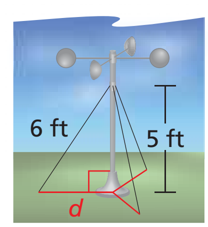
**1.**




**2.**




**3.** An anemometer is a device used to measure wind speed. The anemometer shown is attached to the top of a pole. Support wires are attached to the pole 5 feet above the ground. Each support wire is 6 feet long. How far from the base of the pole is each wire attached to the ground?



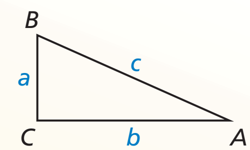
## Using the Converse of the Pythagorean Theorem

The converse of the Pythagorean Theorem is also true. You can use it to determine whether a triangle with given side lengths is a right triangle.

### Theorem

**Theorem 9.2 Converse of the Pythagorean Theorem**

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.



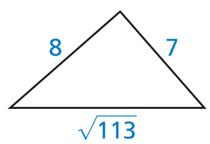

If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.

*Proof* Ex. 39, p. 474

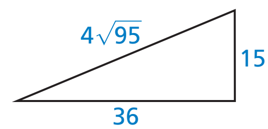
**EXAMPLE 4** **Verifying Right Triangles**

Tell whether each triangle is a right triangle.

**a.**




**b.**




**SELECTING TOOLS**

Use a calculator to determine that $\sqrt{113} \approx 10.630$ is the length of the longest side in part (a).

**SOLUTION**

Let $c$ represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

**a.** $\left(\sqrt{113}\right)^2 \stackrel{?}{=} 7^2 + 8^2$

$113 \stackrel{?}{=} 49 + 64$

$113 = 113$ ✓

▶ The triangle is a right triangle.

**b.** $\left(4\sqrt{95}\right)^2 \stackrel{?}{=} 15^2 + 36^2$

$4^2 \cdot \left(\sqrt{95}\right)^2 \stackrel{?}{=} 15^2 + 36^2$

$16 \cdot 95 \stackrel{?}{=} 225 + 1296$

$1520 \neq 1521$ ✗

▶ The triangle is *not* a right triangle.

### Monitoring Progress

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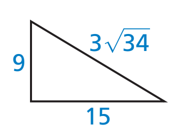
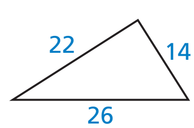
**Tell whether the triangle is a right triangle.**

**4.**




**5.**



## Classifying Triangles

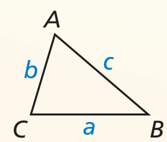
The Converse of the Pythagorean Theorem is used to determine whether a triangle is a right triangle. You can use the theorem below to determine whether a triangle is acute or obtuse.

### Theorem

**Theorem 9.3 Pythagorean Inequalities Theorem**

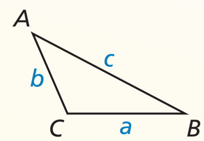
For any $\triangle ABC$, where $c$ is the length of the longest side, the following statements are true.

If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.




$c^2 < a^2 + b^2$

If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.




$c^2 > a^2 + b^2$

*Proof* Exs. 42 and 43, p. 474

**REMEMBER**

The Triangle Inequality Theorem (Theorem 6.11) on page 343 states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

### EXAMPLE 5 Classifying Triangles

Verify that segments with lengths of 4.3 feet, 5.2 feet, and 6.1 feet form a triangle. Is the triangle *acute*, *right*, or *obtuse*?

**SOLUTION**

**Step 1** Use the Triangle Inequality Theorem (Theorem 6.11) to verify that the segments form a triangle.

$4.3 + 5.2 \stackrel{?}{>} 6.1$ $\qquad$ $4.3 + 6.1 \stackrel{?}{>} 5.2$ $\qquad$ $5.2 + 6.1 \stackrel{?}{>} 4.3$

$9.5 > 6.1$ ✓ $\qquad$ $10.4 > 5.2$ ✓ $\qquad$ $11.3 > 4.3$ ✓

▶ The segments with lengths of 4.3 feet, 5.2 feet, and 6.1 feet form a triangle.

**Step 2** Classify the triangle by comparing the square of the length of the longest side with the sum of the squares of the lengths of the other two sides.

| | | |
|---|---|---|
| $c^2$ ▭ $a^2 + b^2$ | | Compare $c^2$ with $a^2 + b^2$. |
| $6.1^2$ ▭ $4.3^2 + 5.2^2$ | | Substitute. |
| $37.21$ ▭ $18.49 + 27.04$ | | Simplify. |
| $37.21 < 45.53$ | | $c^2$ is less than $a^2 + b^2$. |

▶ The segments with lengths of 4.3 feet, 5.2 feet, and 6.1 feet form an acute triangle.

### Monitoring Progress

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**6.** Verify that segments with lengths of 3, 4, and 6 form a triangle. Is the triangle *acute*, *right*, or *obtuse*?

**7.** Verify that segments with lengths of 2.1, 2.8, and 3.5 form a triangle. Is the triangle *acute*, *right*, or *obtuse*?

# 9.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

## Vocabulary and Core Concept Check

1. **VOCABULARY** What is a Pythagorean triple?

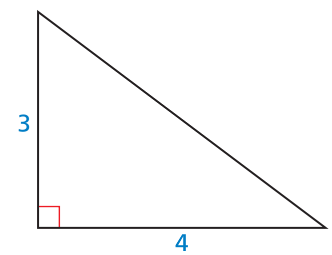
2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

Find the length of the longest side.

Find the length of the hypotenuse.

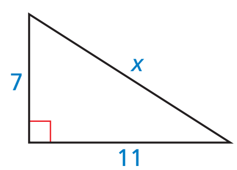
Find the length of the longest leg.

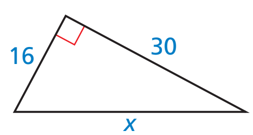
Find the length of the side opposite the right angle.

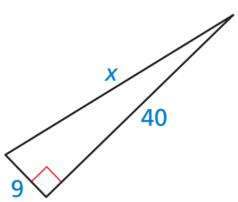


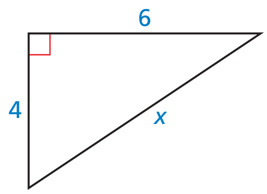

## Monitoring Progress and Modeling with Mathematics

**In Exercises 3–6, find the value of *x*. Then tell whether the side lengths form a Pythagorean triple.** *(See Example 1.)*

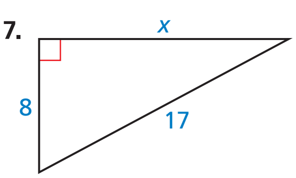
3. 

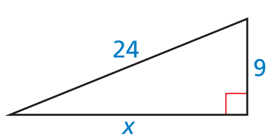

4. 

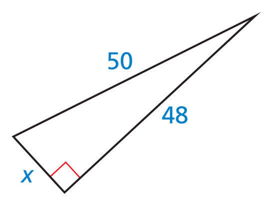

5. 

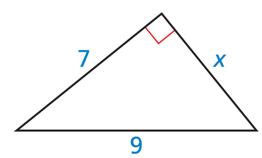

6. 


**In Exercises 7–10, find the value of *x*. Then tell whether the side lengths form a Pythagorean triple.** *(See Example 2.)*

7. 

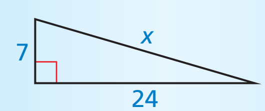

8. 


9. 


10. 

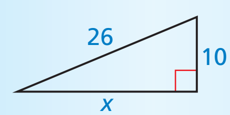

**ERROR ANALYSIS** **In Exercises 11 and 12, describe and correct the error in using the Pythagorean Theorem (Theorem 9.1).**

11. ✗




$$c^2 = a^2 + b^2$$
$$x^2 = 7^2 + 24^2$$
$$x^2 = (7 + 24)^2$$
$$x^2 = 31^2$$
$$x = 31$$

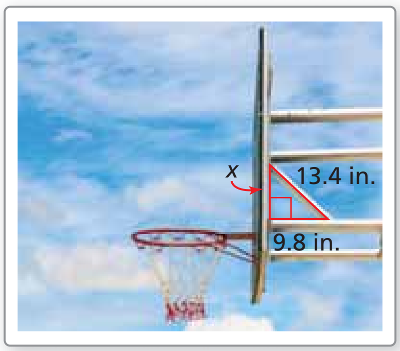
12. ✗




$$c^2 = a^2 + b^2$$
$$x^2 = 10^2 + 26^2$$
$$x^2 = 100 + 676$$
$$x^2 = 776$$
$$x = \sqrt{776}$$
$$x \approx 27.9$$

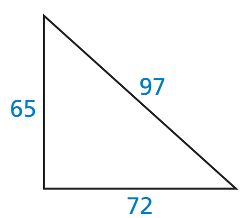
**13.** **MODELING WITH MATHEMATICS** The fire escape forms a right triangle, as shown. Use the Pythagorean Theorem (Theorem 9.1) to approximate the distance between the two platforms. *(See Example 3.)*


**14.** **MODELING WITH MATHEMATICS** The backboard of the basketball hoop forms a right triangle with the supporting rods, as shown. Use the Pythagorean Theorem (Theorem 9.1) to approximate the distance between the rods where they meet the backboard.

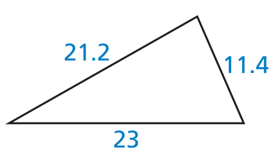



**In Exercises 15–20, tell whether the triangle is a right triangle.** *(See Example 4.)*

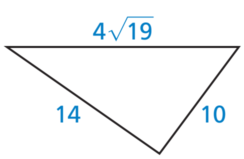
**15.**



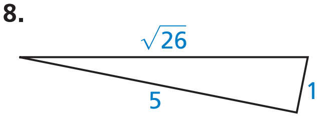

**16.**



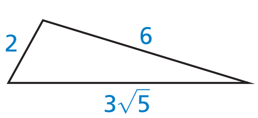

**17.**



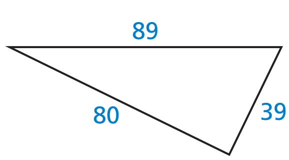

**18.**




**19.**




**20.**




**In Exercises 21–28, verify that the segment lengths form a triangle. Is the triangle *acute*, *right*, or *obtuse*?** *(See Example 5.)*

**21.** 10, 11, and 14

**22.** 6, 8, and 10

**23.** 12, 16, and 20

**24.** 15, 20, and 36

**25.** 5.3, 6.7, and 7.8

**26.** 4.1, 8.2, and 12.2

**27.** 24, 30, and $6\sqrt{43}$

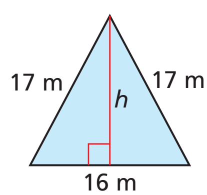
**28.** 10, 15, and $5\sqrt{13}$

**29.** **MODELING WITH MATHEMATICS** In baseball, the lengths of the paths between consecutive bases are 90 feet, and the paths form right angles. The player on first base tries to steal second base. How far does the ball need to travel from home plate to second base to get the player out?

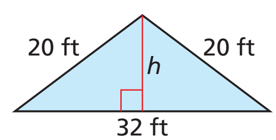
**30.** **REASONING** You are making a canvas frame for a painting using stretcher bars. The rectangular painting will be 10 inches long and 8 inches wide. Using a ruler, how can you be certain that the corners of the frame are 90°?

**In Exercises 31–34, find the area of the isosceles triangle.**

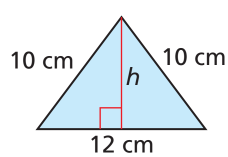
**31.**



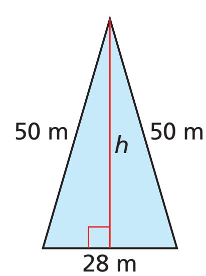

**32.**




**33.**

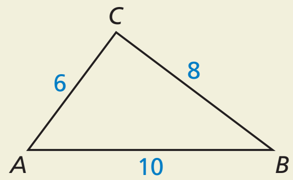



**34.**

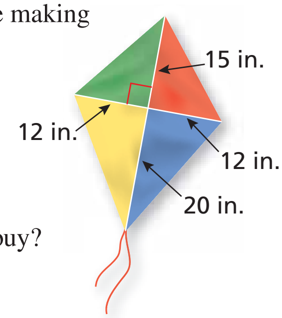


**35. ANALYZING RELATIONSHIPS** Justify the Distance Formula using the Pythagorean Theorem (Thm. 9.1).

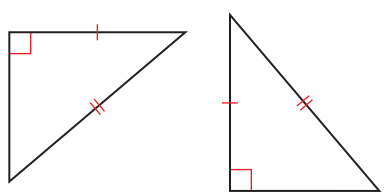
**36. HOW DO YOU SEE IT?** How do you know $\angle C$ is a right angle without using the Pythagorean Theorem (Theorem 9.1)?




**37. PROBLEM SOLVING** You are making a kite and need to figure out how much binding to buy. You need the binding for the perimeter of the kite. The binding comes in packages of two yards. How many packages should you buy?




**38. PROVING A THEOREM** Use the Pythagorean Theorem (Theorem 9.1) to prove the Hypotenuse-Leg (HL) Congruence Theorem (Theorem 5.9).



**39. PROVING A THEOREM** Prove the Converse of the Pythagorean Theorem (Theorem 9.2). (*Hint*: Draw $\triangle ABC$ with side lengths $a$, $b$, and $c$, where $c$ is the length of the longest side. Then draw a right triangle with side lengths $a$, $b$, and $x$, where $x$ is the length of the hypotenuse. Compare lengths $c$ and $x$.)

**40. THOUGHT PROVOKING** Consider two integers $m$ and $n$, where $m > n$. Do the following expressions produce a Pythagorean triple? If yes, prove your answer. If no, give a counterexample.

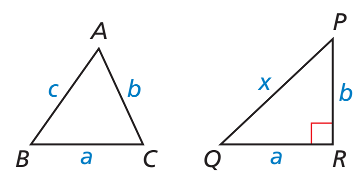
$$2mn,\ m^2 - n^2,\ m^2 + n^2$$

**41. MAKING AN ARGUMENT** Your friend claims 72 and 75 cannot be part of a Pythagorean triple because $72^2 + 75^2$ does not equal a positive integer squared. Is your friend correct? Explain your reasoning.

**42. PROVING A THEOREM** Copy and complete the proof of the Pythagorean Inequalities Theorem (Theorem 9.3) when $c^2 < a^2 + b^2$.

**Given** In $\triangle ABC$, $c^2 < a^2 + b^2$, where $c$ is the length of the longest side.
$\triangle PQR$ has side lengths $a$, $b$, and $x$, where $x$ is the length of the hypotenuse, and $\angle R$ is a right angle.

**Prove** $\triangle ABC$ is an acute triangle.




| STATEMENTS | REASONS |
|---|---|
| **1.** In $\triangle ABC$, $c^2 < a^2 + b^2$, where $c$ is the length of the longest side. $\triangle PQR$ has side lengths $a$, $b$, and $x$, where $x$ is the length of the hypotenuse, and $\angle R$ is a right angle. | **1.** ______________ |
| **2.** $a^2 + b^2 = x^2$ | **2.** ______________ |
| **3.** $c^2 < x^2$ | **3.** ______________ |
| **4.** $c < x$ | **4.** Take the positive square root of each side. |
| **5.** $m\angle R = 90°$ | **5.** ______________ |
| **6.** $m\angle C < m\angle R$ | **6.** Converse of the Hinge Theorem (Theorem 6.13) |
| **7.** $m\angle C < 90°$ | **7.** ______________ |
| **8.** $\angle C$ is an acute angle. | **8.** ______________ |
| **9.** $\triangle ABC$ is an acute triangle. | **9.** ______________ |

**43. PROVING A THEOREM** Prove the Pythagorean Inequalities Theorem (Theorem 9.3) when $c^2 > a^2 + b^2$. (*Hint*: Look back at Exercise 42.)

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Simplify the expression by rationalizing the denominator.** *(Skills Review Handbook)*

**44.** $\frac{7}{\sqrt{2}}$ **45.** $\frac{14}{\sqrt{3}}$ **46.** $\frac{8}{\sqrt{2}}$ **47.** $\frac{12}{\sqrt{3}}$