3.5 Slopes of Lines

Texas Essential Knowledge and Skills
G.2.A
G.2.B

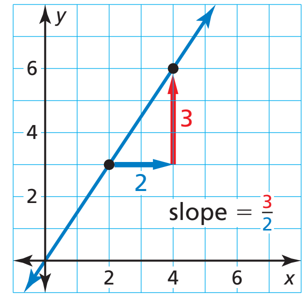
Essential Question

How can you use the slope of a line to describe the line?

Slope is the rate of change between any two points on a line. It is the measure of the *steepness* of the line.

To find the slope of a line, find the ratio of the change in y (vertical change) to the change in x (horizontal change).

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$



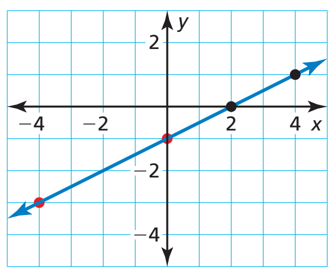
EXPLORATION 1 Finding the Slope of a Line

Work with a partner. Find the slope of each line using two methods.

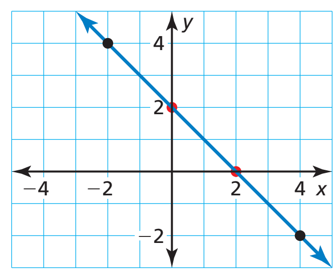
Method 1: Use the two black points. **Method 2:** Use the two pink points

Do you get the same slope using each method? Why do you think this happens?

a.



b.



EXPLORATION 2 Drawing Lines with Given Slopes

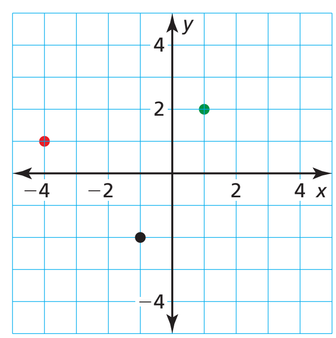
Work with a partner.

a. Draw a line through the black point using a slope of $\frac{3}{4}$. Use the same slope to draw a line through the pink point.

b. Draw a line through the green point using a slope of $-\frac{4}{3}$.

c. What do you notice about the lines through the black and pink points?

d. Describe the angle formed by the lines through the black and green points. What do you notice about the product of the slopes of the two lines?



ANALYZING MATHEMATICAL RELATIONSHIPS

To be proficient in math, you need to look closely to discern a pattern or structure.

Communicate Your Answer

3. How can you use the slope of a line to describe the line?

4. Make a conjecture about two different nonvertical lines in the same plane that have the same slope.

5. Make a conjecture about two lines in the same plane whose slopes have a product of -1.

3.5 Lesson

Core Vocabulary

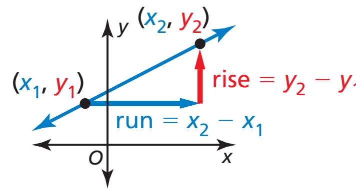
slope, *p. 156*
directed line segment, *p. 157*

What You Will Learn

- Find the slopes of lines
- Use slope to partition directed line segments.
- Identify parallel and perpendicular lines.

Finding the Slopes of Lines

The **slope** of a nonvertical line is the ratio of vertical change (*rise*) to horizontal change (*run*) between any two points on the line. If a line in the coordinate plane passes through points (x_1, y_1) and (x_2, y_2), then the slope m is



$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Core Concept

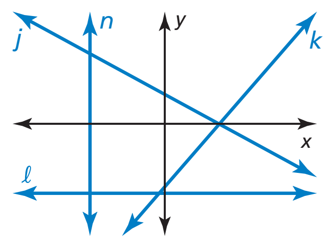
Slopes of Lines in the Coordinate Plane

Negative slope: falls from left to right, as in line j

Positive slope: rises from left to right, as in line k

Zero slope (slope of 0): horizontal, as in line ℓ

Undefined slope: vertical, as in line n

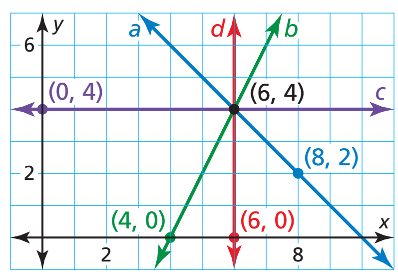


STUDY TIP

When finding slope, you can label either point as (x_1, y_1) and the other point as (x_2, y_2).

EXAMPLE 1 Finding the Slopes of Lines

Find the slopes of lines a, b, c, and d.



SOLUTION

Line a: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{6 - 8} = \frac{2}{-2} = -1$

Line b: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{6 - 4} = \frac{4}{2} = 2$

Line c: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 4}{6 - 0} = \frac{0}{6} = 0$

Line d: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{6 - 6} = \frac{4}{0}$, which is undefined

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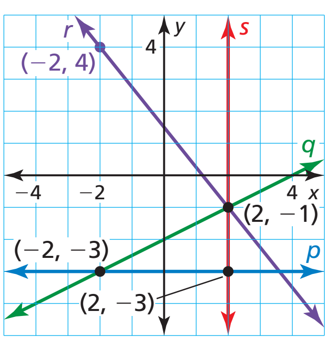


Find the slope of the given line.

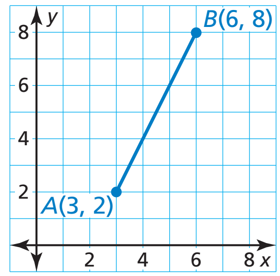
1. line p
2. line q
3. line r
4. line s

Partitioning a Directed Line Segment

A **directed line segment** *AB* is a segment that represents moving from point *A* to point *B*. The following example shows how to use slope to find a point on a directed line segment that partitions the segment in a given ratio.

EXAMPLE 2 Partitioning a Directed Line Segment

Find the coordinates of point *P* along the directed line segment *AB* so that the ratio of *AP* to *PB* is 3 to 2.



SOLUTION

In order to divide the segment in the ratio 3 to 2, think of dividing, or *partitioning*, the segment into 3 + 2, or 5 congruent pieces.

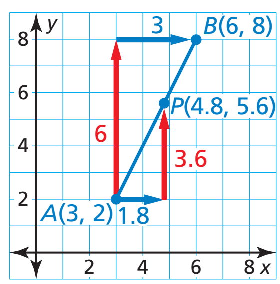
Point *P* is the point that is $\frac{3}{5}$ of the way from point *A* to point *B*.

Find the rise and run from point *A* to point *B*. Leave the slope in terms of rise and run and do not simplify.

$$\text{slope of } \overline{AB}\text{: } m = \frac{8-2}{6-3} = \frac{6}{3} = \frac{\text{rise}}{\text{run}}$$

To find the coordinates of point *P*, add $\frac{3}{5}$ of the run to the *x*-coordinate of *A*, and add $\frac{3}{5}$ of the rise to the *y*-coordinate of *A*.

run: $\frac{3}{5}$ of $3 = \frac{3}{5} \cdot 3 = 1.8$ rise: $\frac{3}{5}$ of $6 = \frac{3}{5} \cdot 6 = 3.6$



▶ So, the coordinates of *P* are

$(3 + 1.8, 2 + 3.6) = (4.8, 5.6).$

The ratio of *AP* to *PB* is 3 to 2.

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Find the coordinates of point *P* along the directed line segment *AB* so that *AP* to *PB* is the given ratio.

5. *A*(l, 3), *B*(8, 4); 4 to 1

6. *A*(−2, 1), *B*(4, 5); 3 to 7

Identifying Parallel and Perpendicular Lines

In the coordinate plane, the *x*-axis and the *y*-axis are perpendicular. Horizontal lines are parallel to the *x*-axis, and vertical lines are parallel to the *y*-axis.

READING

If the product of two numbers is −1, then the numbers are called *negative reciprocals*.

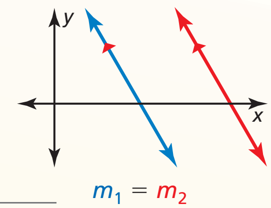
Theorems

Theorem 3.13 Slopes of Parallel Lines

In a coordinate plane, two distinct nonvertical lines are parallel if and only if they have the same slope.

Any two vertical lines are parallel.

Proof p. 443; Ex. 41, p. 448

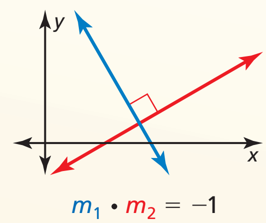


Theorem 3.14 Slopes of Perpendicular Lines

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is −1.

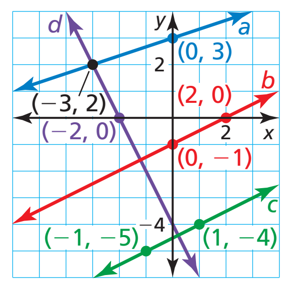
Horizontal lines are perpendicular to vertical lines.

Proof p. 444; Ex. 42, p. 448



EXAMPLE 3 Identifying Parallel and Perpendicular Lines

Determine which of the lines are parallel and which of the lines are perpendicular.



SOLUTION

Find the slope of each line.

$$\text{Line } a\text{: } m = \frac{3-2}{0-(-3)} = \frac{1}{3}$$

$$\text{Line } b\text{: } m = \frac{0-(-1)}{2-0} = \frac{1}{2}$$

$$\text{Line } c\text{: } m = \frac{-4-(-5)}{1-(-1)} = \frac{1}{2}$$

$$\text{Line } d\text{: } m = \frac{2-0}{-3-(-2)} = -2$$

Because lines *b* and *c* have the same slope, lines *b* and *c* are parallel. Because $\frac{1}{2}(-2) = -1$, lines *b* and *d* are perpendicular and lines *c* and *d* are perpendicular.

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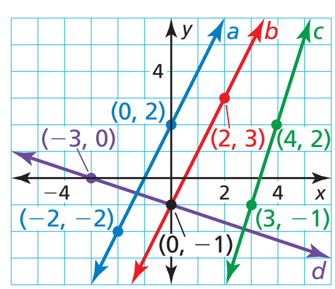
7. Determine which of the lines are parallel and which of the lines are perpendicular.



3.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

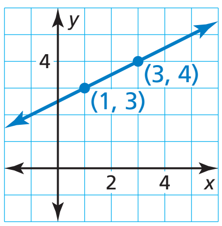
1. **COMPLETE THE SENTENCE** A ________ line segment *AB* is a segment that represents moving from point *A* to point *B*.

2. **WRITING** How are the slopes of perpendicular lines related?

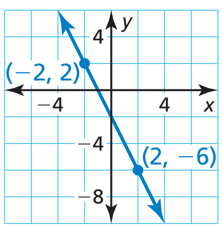
Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, find the slope of the line that passes through the given points. *(See Example 1.)*

3.



4.



5. $(-5, -1), (3, -1)$

6. $(2, 1), (0, 6)$

7. $(-1, -4), (1, 2)$

8. $(-7, 0), (-7, -6)$

In Exercises 9–12, graph the line through the given point with the given slope.

9. $P(3, -2), m = -\frac{1}{6}$

10. $P(-4, 0), m = \frac{5}{2}$

11. $P(0, 5), m = \frac{2}{3}$

12. $P(2, -6), m = -\frac{7}{4}$

In Exercises 13–16, find the coordinates of point *P* along the directed line segment *AB* so that *AP* to *PB* is the given ratio. *(See Example 2.)*

13. $A(8, 0), B(3, -2)$; 1 to 4

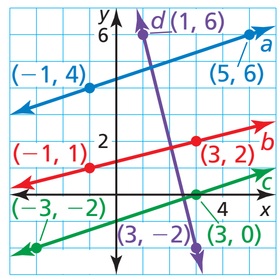
14. $A(-2, -4), B(6, 1)$; 3 to 2

15. $A(1, 6), B(-2, -3)$; 5 to 1

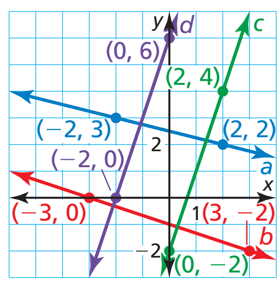
16. $A(-3, 2), B(5, -4)$; 2 to 6

In Exercises 17 and 18, determine which of the lines are parallel and which of the lines are perpendicular. *(See Example 3.)*

17.



18.



In Exercises 19–22, tell whether the lines through the given points are *parallel*, *perpendicular*, or *neither*. Justify your answer.

19. **Line 1:** $(1, 0), (7, 4)$
Line 2: $(7, 0), (3, 6)$

20. **Line 1:** $(-3, 1), (-7, -2)$
Line 2: $(2, -1), (8, 4)$

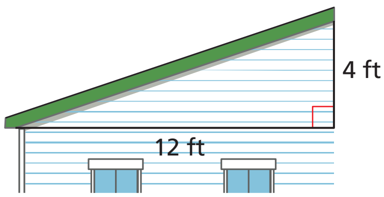
21. **Line 1:** $(-9, 3), (-5, 7)$
Line 2: $(-11, 6), (-7, 2)$

22. **Line 1:** $(10, 5), (-8, 9)$
Line 2: $(2, -4), (11, -6)$

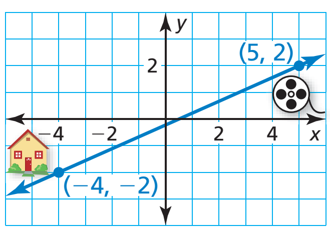
23. **ERROR ANALYSIS** Describe and correct the error in determining whether the lines are parallel, perpendicular, or neither.

> ✗
> Line 1: (3, −5), (2, −1)
> Line 2: (0, 3), (1, 7)
> $m_1 = \frac{-1-(-5)}{2-3} = -4 \quad m_2 = \frac{7-3}{1-0} = 4$
> Lines 1 and 2 are perpendicular.

24. **MODELING WITH MATHEMATICS** Carpenters refer to the slope of a roof as the *pitch* of the roof. Find the pitch of the roof.



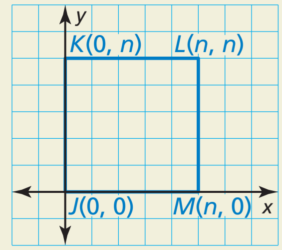
25. **MODELING WITH MATHEMATICS** Your school lies directly between your house and the movie theater. The distance from your house to the school is one-fourth of the distance from the school to the movie theater. What point on the graph represents your school?



26. **ABSTRACT REASONING** Make a conjecture about how to find the coordinates of a point that lies beyond point B along $\overrightarrow{AB}$. Use an example to support your conjecture.

27. **CRITICAL THINKING** Suppose point P divides the directed line segment XY so that the ratio of XP to PY is 3 to 5. Describe the point that divides the directed line segment YX so that the ratio of YP to PX is 5 to 3.

28. **HOW DO YOU SEE IT?** Determine whether quadrilateral $JKLM$ is a square. Explain your reasoning.



29. **WRITING** Explain how to determine which of two lines is steeper without graphing them.

30. **THOUGHT PROVOKING** Describe a real-life situation that can be modeled by parallel lines. Explain how you know that the lines would be parallel.

31. **REASONING** A triangle has vertices $L(0, 6)$, $M(5, 8)$, and $N(4, -1)$. Is the triangle a right triangle? Explain your reasoning.

PROVING A THEOREM In Exercises 32 and 33, use the slopes of lines to write a paragraph proof of the theorem.

32. Lines Perpendicular to a Transversal Theorem (Theorem 3.12): In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

33. Transitive Property of Parallel Lines Theorem (Theorem 3.9): If two lines are parallel to the same line, then they are parallel to each other.

34. **PROOF** Prove the statement: If two lines are vertical, then they are parallel.

35. **PROOF** Prove the statement: If two lines are horizontal, then they are parallel.

36. **PROOF** Prove that horizontal lines are perpendicular to vertical lines.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Identify the slope and the *y*-intercept of the line. *(Skills Review Handbook)*

37. $y = 3x + 9$

38. $y = -\frac{1}{2}x + 7$

39. $y = \frac{1}{6}x - 8$

40. $y = -8x - 6$