3.6 Equations of Parallel and Perpendicular Lines

Texas Essential Knowledge and Skills

G.2.B
G.2.C

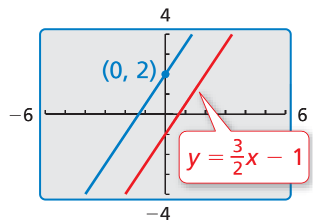
Essential Question

How can you write an equation of a line that is parallel or perpendicular to a given line and passes through a given point?

EXPLORATION 1 Writing Equations of Parallel and Perpendicular Lines

Work with a partner. Write an equation of the line that is parallel or perpendicular to the given line and passes through the given point. Use a graphing calculator to verify your answer.

a.



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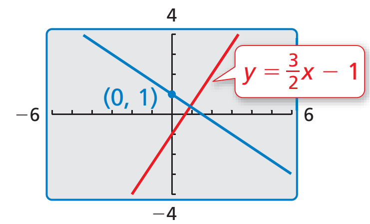

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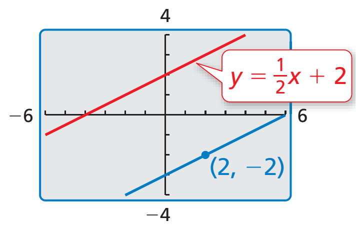

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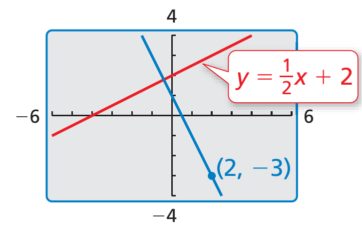

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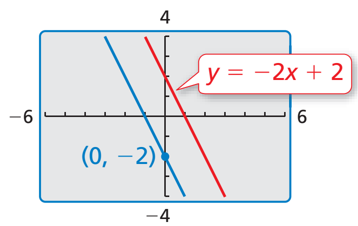

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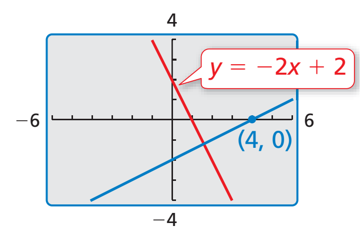
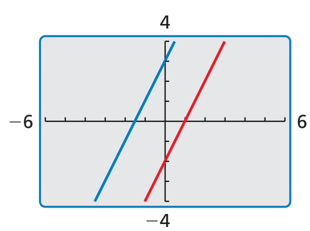


EXPLORATION 2 Writing Equations of Parallel and Perpendicular Lines

Work with a partner. Write the equations of the parallel or perpendicular lines. Use a graphing calculator to verify your answers.

a.



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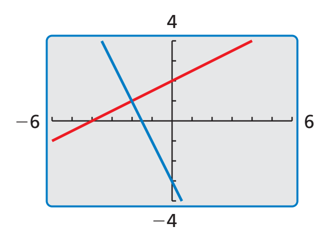


APPLYING MATHEMATICS

To be proficient in math, you need to analyze relationships mathematically to draw conclusions.

Communicate Your Answer

3. How can you write an equation of a line that is parallel or perpendicular to a given line and passes through a given point?

4. Write an equation of the line that is (a) parallel and (b) perpendicular to the line $y = 3x + 2$ and passes through the point $(1, -2)$.

3.6 Lesson

What You Will Learn

- Write equations of parallel and perpendicular lines.
- Use slope to find the distance from a point to a line.

Core Vocabulary

Previous
slope-intercept form
y-intercept

Writing Equations of Parallel and Perpendicular Lines

You can apply the Slopes of Parallel Lines Theorem (Theorem 3.13) and the Slopes of Perpendicular Lines Theorem (Theorem 3.14) to write equations of parallel and perpendicular lines.

EXAMPLE 1 **Writing an Equation of a Parallel Line**

Write an equation of the line passing through the point $(-1, 1)$ that is parallel to the line $y = 2x - 3$.

REMEMBER
The linear equation $y = 2x - 3$ is written in slope-intercept form $y = mx + b$, where m is the slope and b is the y-intercept.

SOLUTION

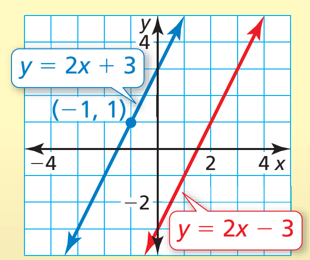
Step 1 Find the slope m of the parallel line. The line $y = 2x - 3$ has a slope of 2. By the Slopes of Parallel Lines Theorem (Theorem 3.13), a line parallel to this line also has a slope of 2. So, $m = 2$.

Step 2 Find the y-intercept b by using $m = 2$ and $(x, y) = (-1, 1)$.

$y = mx + b$	Use slope-intercept form.
$1 = 2(-1) + b$	Substitute for m, x, and y.
$3 = b$	Solve for b.

- Because $m = 2$ and $b = 3$, an equation of the line is $y = 2x + 3$. Use a graph to check that the line $y = 2x - 3$ is parallel to the line $y = 2x + 3$.

Check



Monitoring Progress

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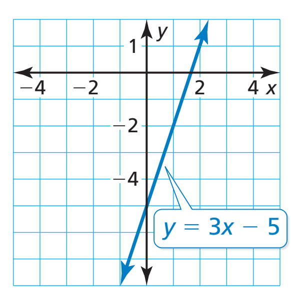
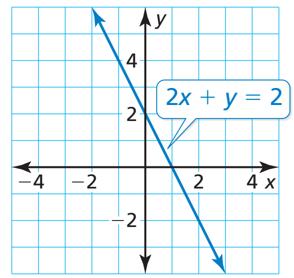
1. Write an equation of the line that passes through the point (1, 5) and is parallel to the given line. Graph the equations of the lines to check that they are parallel.



EXAMPLE 2 **Writing an Equation of a Perpendicular Line**

Write an equation of the line passing through the point (2, 3) that is perpendicular to the given line.



SOLUTION

Step 1 Find the slope m of the perpendicular line. The line $2x + y = 2$, or $y = -2x + 2$, has a slope of -2. Use the Slopes of Perpendicular Lines Theorem (Theorem 3.14).

$-2 \cdot m = -1$ — The product of the slopes of ⊥ lines is −1.

$m = \frac{1}{2}$ — Divide each side by −2.

Step 2 Find the y-intercept b by using $m = \frac{1}{2}$ and $(x, y) = (2, 3)$.

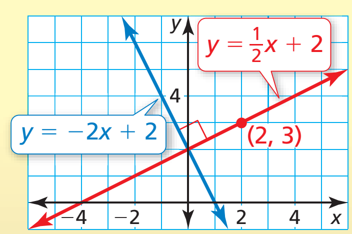
$y = mx + b$ — Use slope-intercept form.

$3 = \frac{1}{2}(2) + b$ — Substitute for m, x, and y.

$2 = b$ — Solve for b.

▶ Because $m = \frac{1}{2}$ and $b = 2$, an equation of the line is $y = \frac{1}{2}x + 2$. Check that the lines are perpendicular by graphing their equations and using a protractor to measure one of the angles formed by their intersection.

Check



Monitoring Progress

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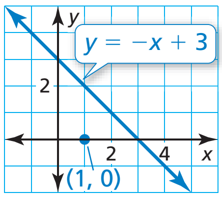
2. Write an equation of the line that passes through the point $(-2, 7)$ and is perpendicular to the line $y = 2x + 9$. Graph the equations of the lines to check that they are perpendicular.

3. How do you know that the lines $x = 4$ and $y = 2$ are perpendicular?

Finding the Distance from a Point to a Line

Recall that the distance from a point to a line is the length of the perpendicular segment from the point to the line.

EXAMPLE 3 Finding the Distance from a Point to a Line



Find the distance from the point (1, 0) to the line $y = -x + 3$.

SOLUTION

Step 1 Find the equation of the line perpendicular to the line $y = -x + 3$ that passes through the point (1, 0).

First, find the slope m of the perpendicular line. The line $y = -x + 3$ has a slope of -1. Use the Slopes of Perpendicular Lines Theorem (Theorem 3.14).

$-1 \cdot m = -1$ — The product of the slopes of ⊥ lines is -1.

$m = 1$ — Divide each side by -1.

Then find the y-intercept b by using $m = 1$ and $(x, y) = (1, 0)$.

$y = mx + b$ — Use slope-intercept form.

$0 = 1(1) + b$ — Substitute for x, y, and m.

$-1 = b$ — Solve for b.

Because $m = 1$ and $b = -1$, an equation of the line is $y = x - 1$.

REMEMBER

Recall that the solution of a system of two linear equations in two variables gives the coordinates of the point of intersection of the graphs of the equations.

There are two special cases when the lines have the same slope.

- When the system has no solution, the lines are parallel.
- When the system has infinitely many solutions, the lines coincide.

Step 2 Use the two equations to write and solve a system of equations to find the point where the two lines intersect.

$y = -x + 3$ — Equation 1

$y = x - 1$ — Equation 2

Substitute $-x + 3$ for y in Equation 2.

$y = x - 1$ — Equation 2

$-x + 3 = x - 1$ — Substitute $-x + 3$ for y.

$x = 2$ — Solve for x.

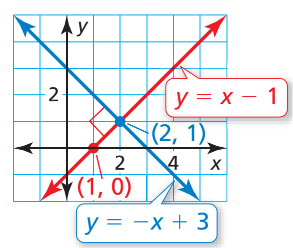
Substitute 2 for x in Equation 1 and solve for y.

$y = -x + 3$ — Equation 1

$y = -2 + 3$ — Substitute 2 for x.

$y = 1$ — Simplify.

So, the perpendicular lines intersect at (2, 1).



Step 3 Use the Distance Formula to find the distance from (1, 0) to (2, 1).

$$\text{distance} = \sqrt{(1 - 2)^2 + (0 - 1)^2} = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2} \approx 1.4$$

▶ So, the distance from the point (1, 0) to the line $y = -x + 3$ is about 1.4 units.

Monitoring Progress

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4. Find the distance from the point (6, 4) to the line $y = x + 4$.

5. Find the distance from the point $(-1, 6)$ to the line $y = -2x$.

3.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** To find the distance from point P to line g, you must first find the equation of the line ________ to line g that passes through point P.

2. **WRITING** Explain how to write an equation of the line that passes through the point (3, 1) and is (a) parallel to the line $y = 5$ and (b) perpendicular to the line $y = 5$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, write an equation of the line passing through point P that is parallel to the given line. Graph the equations of the lines to check that they are parallel. *(See Example 1.)*

3. $P(0, -1)$, $y = -2x + 3$

4. $P(3, 8)$, $y = \frac{1}{5}(x + 4)$

5. $P(-2, 6)$, $x = -5$

6. $P(4, 0)$, $-x + 2y = 12$

In Exercises 7–10, write an equation of the line passing through point P that is perpendicular to the given line. Graph the equations of the lines to check that they are perpendicular. *(See Example 2.)*

7. $P(0, 0)$, $y = -9x - 1$

8. $P(4, -6)$, $y = -3$

9. $P(2, 3)$, $y - 4 = -2(x + 3)$

10. $P(-8, 0)$, $3x - 5y = 6$

In Exercises 11–14, find the distance from point A to the given line. *(See Example 3.)*

11. $A(-1, 7)$, $y = 3x$

12. $A(-9, -3)$, $y = x - 6$

13. $A(15, -21)$, $5x + 2y = 4$

14. $A\left(-\frac{1}{4}, 5\right)$, $-x + 2y = 14$

15. **ERROR ANALYSIS** Describe and correct the error in writing an equation of the line that passes through the point (3, 4) and is parallel to the line $y = 2x + 1$.

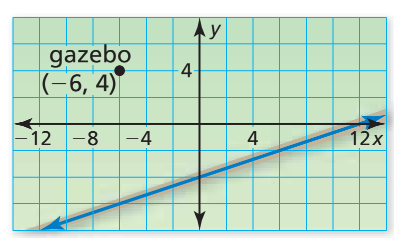
$$y = 2x + 1, (3, 4)$$
$$4 = m(3) + 1$$
$$1 = m$$

The line $y = x + 1$ is parallel to the line $y = 2x + 1$.

16. **MODELING WITH MATHEMATICS** A new road is being constructed parallel to train tracks through point $V(3, -2)$. An equation of the line representing the train tracks is $y = 2x$. Find an equation of the line representing the new road.

17. **MODELING WITH MATHEMATICS** A bike path is being constructed perpendicular to Washington Boulevard through point $P(2, 2)$. An equation of the line representing Washington Boulevard is $y = -\frac{2}{3}x$. Find an equation of the line representing the bike path.

18. **PROBLEM SOLVING** A gazebo is being built near a nature trail. An equation of the line representing the nature trail is $y = \frac{1}{3}x - 4$. Each unit in the coordinate plane corresponds to 10 feet. Approximately how far is the gazebo from the nature trail?



In Exercises 19–22, find the midpoint of $\overline{PQ}$. Then write an equation of the line that passes through the midpoint and is perpendicular to $\overline{PQ}$. This line is called the *perpendicular bisector*.

19. $P(-4, 3), Q(4, -1)$

20. $P(-5, -5), Q(3, 3)$

21. $P(0, 2), Q(6, -2)$

22. $P(-7, 0), Q(1, 8)$

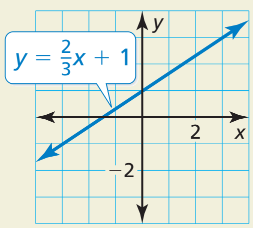
23. MODELING WITH MATHEMATICS Two cars are traveling at a speed of 30 miles per hour. The second car is 3 miles ahead of the first car.

a. Write and graph a linear equation that represents the position p of the first car after t hours.

b. Write and graph a linear equation that represents the position p of the second car after t hours.

c. Are the lines in parts (a) and (b) *parallel*, *perpendicular*, or *neither*? Explain your reasoning.

24. HOW DO YOU SEE IT? Determine whether each equation is the equation of a line *parallel* to the given line, *perpendicular* to the given line, or *neither*. Explain your reasoning.



a. $y = \frac{2}{3}x - 1$

b. $y = \frac{3}{2}x + 3$

c. $y = -\frac{2}{3}x + 2$

d. $y = -\frac{3}{2}x$

25. MAKING AN ARGUMENT Your classmate claims that no two nonvertical parallel lines can have the same y-intercept. Is your classmate correct? Explain.

26. MATHEMATICAL CONNECTIONS Solve each system of equations algebraically. Make a conjecture about what the solution(s) can tell you about whether the lines intersect, are parallel, or are the same line.

a. $y = 4x + 9$
$4x - y = 1$

b. $3y + 4x = 16$
$2x - y = 18$

c. $y = -5x + 6$
$10x + 2y = 12$

MATHEMATICAL CONNECTIONS **In Exercises 27 and 28, find a value for k based on the given description.**

27. The line through $(-1, k)$ and $(-7, -2)$ is parallel to the line $y = x + 1$.

28. The line through $(k, 2)$ and $(7, 0)$ is perpendicular to the line $y = x - \frac{28}{5}$.

29. PROBLEM SOLVING What is the distance between the lines $y = 2x$ and $y = 2x + 5$? Verify your answer.

30. THOUGHT PROVOKING Find a formula for the distance from the point (x_0, y_0) to the line $ax + by = 0$. Verify your formula using a point and a line.

31. COMPARING METHODS The point (x, y) lies on the line $y = x + 2$. The distance from $P(1, 0)$ to (x, y) is represented by d.

a. Write d as an expression in terms of x.

b. Explain how you can use the expression from part (a) to find the shortest distance from point P to the line $y = x + 2$.

c. Compare this method to the method used in Example 3. Which method do you prefer? Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Plot the point in a coordinate plane. *(Skills Review Handbook)*

32. $A(3, 6)$

33. $B(0, -4)$

34. $C(5, 0)$

35. $D(-1, -2)$

Copy and complete the table. *(Skills Review Handbook)*

36.

x	-2	-1	0	1	2
$y = x + 9$					

37.

x	-2	-1	0	1	2
$y = x - \frac{3}{4}$					