Chapter 2

2.2 Exercises (pp. 80–82)

Vocabulary and Core Concept Check

1. A postulate is a rule that is accepted without proof. A conjecture is an unproven statement based on observation. So, a conjecture is something you notice that seems to be true, and you are going to investigate further to test it out and/or try to prove it. Postulates, however, are obviously true without a doubt, and they are used to prove the most basic and fundamental ideas of geometry.

2. Inductive reasoning uses patterns to write a conjecture. Deductive reasoning uses facts, definitions, accepted properties, and the laws of logic to form a logical argument.

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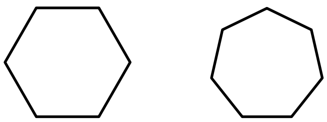
3. The absolute value of each number in the list is 1 greater than the absolute value of the previous number in the list, and the signs alternate from positive to negative. The next two numbers are: $-6, 7$.

4. The numbers are increasing by successive multiples of 2. The sequence is: $0 + 2 = 2, 2 + 4 = 6, 6 + 6 = 12,$ $12 + 8 = 20, 20 + 10 = 30, 30 + 12 = 42$, etc. So, the next two numbers are: 30, 42.

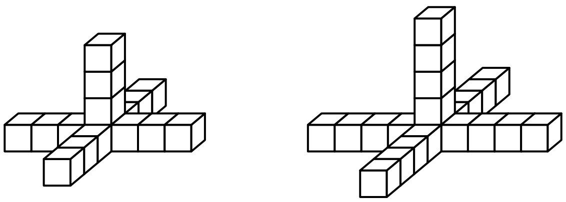
5. The pattern is the alphabet written backward. The next two letters are: U, T.

6. The letters represent the first letter of each month of the year, and they are in the order of the months. The next two letters are: J, J.

7. The pattern is regular polygons having one more side than the previous polygon.



8. The pattern is the addition of 5 blocks to the previous figure. One block is added to each of the four ends of the base and one block is added on top. So, the next two figures will have 16 blocks and then 21 blocks.



9. The product of any two even integers is an even integer.

Tests: $2 \cdot 8 = 16, \quad 22 \cdot 20 = 440$

10. The sum of an even integer and an odd integer is an odd integer.

Tests: $3 + 4 = 7, \quad 6 + 13 = 19$

11. The quotient of a number and its reciprocal is the square of that number.

Tests: $\dfrac{10}{\left(\frac{1}{10}\right)} = \dfrac{10}{1} \cdot \dfrac{10}{1} = 100 = 10^2$

$$\frac{\left(\frac{2}{3}\right)}{\left(\frac{3}{2}\right)} = \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9} = \left(\frac{2}{3}\right)^2$$

12. The quotient of two negative numbers is a positive rational number.

Tests: $\dfrac{-24}{-12} = 2, \quad \dfrac{-33}{-3} = 11$

13. *Sample answer:* Let the two positive numbers be $\frac{1}{2}$ and $\frac{1}{6}$.

The product is $\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$. Because $\frac{1}{12} < \frac{1}{2}$ and $\frac{1}{12} < \frac{1}{6}$, the product of two positive numbers is not always greater than either number.

14. *Sample answer:* Let $n = -1$.

$$\frac{-1 + 1}{-1} = 0$$

$$0 \ngtr 1$$

15. Each angle could be 90°. Then neither are acute.

16. If line s intersects $\overline{MN}$ at any point other than the midpoint, it is not a segment bisector.

17. You passed the class.

18. not possible; You may get to the movies by other means.

19. not possible; $QRST$ could be a rectangle.

20. P is the midpoint of $\overline{LH}$.

21. not possible

22. If $\frac{1}{2}a = 1\frac{1}{2}$, then $5a = 15$.

23. If a figure is a rhombus, then the figure has two pairs of opposite sides that are parallel.

24. not possible

25. The law of logic used was the Law of Syllogism.

26. The law of logic used was the Law of Detachment.

27. The law of logic used was the Law of Detachment.

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28. The law of logic used was Law of Syllogism.

29. $1 + 3 = 4, \quad 3 + 5 = 8, \quad 7 + 9 = 16$

Conjecture: The sum of two odd integers is an even integer. Let m and n be integers, then $(2m + 1)$ and $(2n + 1)$ are odd integers.

$$\begin{aligned}(2m + 1) + (2n + 1) &= 2m + 2n + 2 \\ &= 2(m + n + 1)\end{aligned}$$

Any number multiplied by 2 is an even number. Therefore, the sum of two odd integers is an even integer.

30. $1 \cdot 3 = 3, \quad 3 \cdot 5 = 15, \quad 7 \cdot 9 = 63$

Conjecture: The product of two odd integers is an odd integer. Let m and n be integers. Then $(2m + 1)$ and $(2n + 1)$ are odd integers.

$$\begin{aligned}(2m + 1)(2n + 1) &= 4mn + 2m + 2n + 1 \\ &= 2(2mn + m + n) + 1\end{aligned}$$

Any number multiplied by 2 is an even number, and adding 1 will yield an odd number. Therefore, the product of two odd integers is an odd integer.

31. inductive reasoning; The conjecture is based on the assumption that a pattern, observed in specific cases, will continue.

32. deductive reasoning; The conclusion is based on mathematical definitions and properties.

33. deductive reasoning; Laws of nature and the Law of Syllogism were used to draw the conclusion.

34. inductive reasoning; The conjecture is based on the assumption that a pattern, observed in specific cases, will continue.

35. The Law of Detachment cannot be used because the hypothesis is not true; *Sample answer:* Using the Law of Detachment, because a square is a rectangle, you can conclude that a square has four sides.

36. The conjecture was based on a pattern in specific cases, not rules or laws about the general case; Using inductive reasoning, you can make a conjecture that you will arrive at school before your friend tomorrow.

37. Using inductive reasoning, you can make a conjecture that male tigers weigh more than female tigers because this was true in all of the specific cases listed in the table.

38. **a.** yes; Bases on inductive reasoning, the pattern in all of the years shown is that the number of girls participating is more than the year before.

b. no; There is no information in the graph about how the number of girl participants compares with the number of boy participants.

39. 1: $2 = 1(2)$

2: $2 + 4 = 6 = 2(3)$

3: $2 + 4 + 6 = 12 = 3(4)$

4: $2 + 4 + 6 + 8 = 20 = 4(5)$

5: $2 + 4 + 6 + 8 + 10 = 30 = 5(6)$

$\vdots \qquad \vdots$

n: $n(n + 1)$

So, the sum of the first n positive even integers is $n(n + 1)$.

40. **a.** $1 + 1 = 2, 2 + 1 = 3, 3 + 2 = 5, 5 + 3 = 8,$
$8 + 5 = 13, 13 + 8 = 21, 21 + 13 = 34$

Each number in the sequence is the sum of the previous two numbers in the sequence.

b. $21 + 34 = 55$

$34 + 55 = 89$

$55 + 89 = 144$

c. *Sample answer:* A spiral can be drawn by connecting the opposite corners of squares with side lengths that follow the Fibonacci sequence. This spiral is similar to the spiral seen on nautilus shells. It is also similar to the golden spiral, which is sometimes found in spiraling galaxies.

41. Argument 2: This argument uses the Law of Detachment to say that when the hypothesis is met, the conclusion is true.

42. Pattern 1: Multiply each term by 2.

$\frac{1}{4} \cdot 2 = \frac{1}{2}, \ \frac{1}{2} \cdot 2 = \frac{2}{2} = 1, \ 1 \cdot 2 = 2, \ 2 \cdot 2 = 4, \ 2 \cdot 4 = 8$

Pattern 2: Add $\frac{1}{4}$ to the previous term.

$\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

$\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

$\frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

$1 + \frac{1}{4} = \frac{4}{4} + \frac{1}{4} = \frac{5}{4}$

$\frac{5}{4} + \frac{1}{4} = \frac{6}{4} = \frac{3}{2}$

Pattern 3: Multiply each term by half the reciprocal of the previous term.

$\frac{1}{4} \cdot \left(\frac{1}{2} \cdot 4\right) = \frac{1}{4} \cdot 2 = \frac{1}{2}$

$\frac{1}{2} \cdot \left(\frac{1}{2} \cdot 2\right) = \frac{1}{2} \cdot 2 = \frac{1}{2}$

$\frac{1}{2} \cdot \left(\frac{1}{2} \cdot 2\right) = \frac{1}{2} \cdot 2 = \frac{1}{2}$

$\frac{1}{2} \cdot \left(\frac{1}{2} \cdot 2\right) = \frac{1}{2} \cdot 2 = \frac{1}{2}$

43. The value of y is 2 more than three times the value of x;
$y = 3x + 2$;
Sample answer: If $x = 10$, then $y = 3(10) + 2 = 32$;
If $x = 72$, then $y = 3(72) + 2 = 218$.

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44. a. Figure 1 has a perimeter of 4.
Figure 2 has a perimeter of 8.
Figure 3 has a perimeter of 12.
Figure 4 has a perimeter of 16.
Figure 5 has a perimeter of 20.
Figure 6 has a perimeter of 24.
Figure 7 has a perimeter of 28.
The perimeter is equal to the product of 4 and the figure number.

b. The 20th figure has a perimeter of $4(20) = 80$.

45. a. true; Based on the Law of Syllogism, if you went camping at Yellowstone, and Yellowstone is in Wyoming, then you went camping in Wyoming.

b. false; When you go camping, you go canoeing, but even though your friend always goes camping when you do, he or she may not choose to go canoeing with you.

c. true; We know that if you go on a hike, your friend goes with you, and we know that you went on a hike. So, based on the Law of Detachment, your friend went on a hike.

d. false; We know that you and your friend went on a hike, but we do not know where. We just know that there is a 3-mile long trail near where you went camping.

46. a. Mineral C must be Talc. Because it was scratched by all three of the other minerals, it must have the lowest hardness rating. Because Mineral B has a higher hardness rating than Mineral A, Mineral A could be either Gypsum or Calcite, and Mineral B could be either Calcite or Fluorite.

b. Check Mineral B and Mineral D. If Mineral D scratches Mineral B, then Mineral D is Fluorite, Mineral B is Calcite, and Mineral A is Gypsum. If Mineral B scratches Mineral D, then Mineral B is Fluorite, and you have to check Mineral D and Mineral A. The one that scratches the other has the higher hardness rating and is therefore Calcite. The one that gets scratched is Gypsum.

Maintaining Mathematical Proficiency

47. Segment Addition Postulate (Post. 1.2)

48. Angle Addition Postulate (Post. 1.4)

49. Ruler Postulate (Post. 1.1)

50. Protractor Postulate (Post. 1.3)

2.3 Explorations (p. 83)

1. The diagram can be turned at any angle to the right or to the left and the lines will appear to be perpendicular.

2. a. true; For every set of two intersecting lines, there is exactly one plane that is defined, so it can be assumed that all of the points shown are coplanar.

b. false; For every two points there is exactly one line, the third point does not necessarily have to be on the same line as the other two.

c. true; All three points lie on the same line, $\overleftrightarrow{AH}$.

d. true; $\angle GFH$ is marked as a right angle.

e. true; By definition of a linear pair, the sides of $\angle BCA$ and $\angle ACD$ form a straight line (straight angle).

f. false; $\overleftrightarrow{AF}$ and $\overleftrightarrow{BD}$ are not necessarily perpendicular because the angle is not marked.

g. false; $\overleftrightarrow{EG}$ and $\overleftrightarrow{BD}$ are not necessarily parallel, there is not enough information about the related angles.

h. true; $\overleftrightarrow{AF}$ and $\overleftrightarrow{BD}$ are coplanar.

i. false; $\overleftrightarrow{EG}$ and $\overleftrightarrow{BD}$ could possibly intersect.

j. true; $\overleftrightarrow{AF}$ and $\overleftrightarrow{BD}$ intersect at point C.

k. false; $\overleftrightarrow{EG}$ and $\overleftrightarrow{AH}$ are perpendicular. So, $\overleftrightarrow{EG}$ cannot be perpendicular to two different lines that intersect.

l. true; $\angle ACD$ and $\angle BCF$ form two pairs of opposite rays.

m. true; $\overleftrightarrow{AC}$ and $\overleftrightarrow{FH}$ are the same line because the points A, C, F, and H are all collinear.

3. You can assume intersecting lines, opposite rays, vertical angles, linear pairs, adjacent angles, coplanar (points, lines, rays, etc.), collinear points, which point is between two other points, and which points are in the interior of an angle. You have to have a label for identifying angle measures, segment lengths, perpendicular lines, parallel lines, and congruent segments or angles.

4. *Sample answer:* $\angle ACD$ and $\angle DCF$ form a linear pair, because these angles share a vertex and a side but no common interior points and $\angle ACF$ is a straight angle. $\angle CFE$ and $\angle GFH$ are vertical angles, because $\overrightarrow{FG}$ and $\overrightarrow{FE}$ are opposite rays as well as $\overrightarrow{FC}$ and $\overrightarrow{FH}$; $\angle DCF$ is a right angle, which cannot be assumed because angle measurements have to be marked. $\overline{BC} \cong \overline{CD}$, which cannot be assumed because lengths of segments have to be labeled.

2.3 Monitoring Progress (pp. 85–86)

1. Plane Intersection Postulate (Post. 2.7)

2. a. Line n passes through points A and B.

b. Line n contains points A and B.

c. Line m and line n intersect at point A.

3. Mark each segment with double tick marks to show that $\overline{PW} \cong \overline{WQ}$.

4. *Sample answer:* $\angle TWP$ and $\angle PWV$ are supplementary.

5. Yes, by the Plane Intersection Postulate (Post. 2.7), plane T intersects plane S at $\overleftrightarrow{BC}$.

6. Because of the right angle symbol you know that plane T is perpendicular to plane S. If $\overleftrightarrow{AB}$ is perpendicular to plane S and $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{BC}$ in plane S at point B, then $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$.

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2.3 Exercises (pp. 87–88)

Vocabulary and Core Concept Check

1. Through any three noncollinear points, there exists exactly one plane.

2. Two points determine a line, which could be on infinitely many planes, but only one plane will go through those two points and a third noncollinear point.

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3. Two Point Postulate (Post. 2.1): Through any two points there exists exactly one line.

4. Plane-Point Postulate (Post. 2.5): A plane contains at least three noncollinear points.

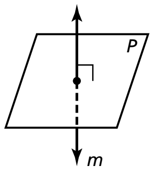
5. *Sample answer:* Line p contains points H and G.

6. *Sample answer:* Lines p and q intersect at point H.

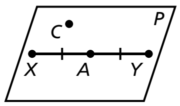
7. *Sample answer:* Through points J, G, and L there is exactly one plane, which is plane M.

8. *Sample answer:* Points J and K lie in plane M, so line q lies in plane M.

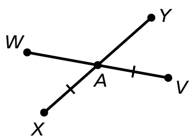
9. Plane P and line m intersect at a 90° angle.



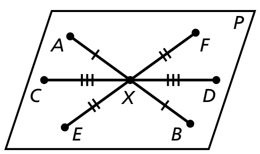
10. Plane P contains $\overline{XY}$, point A bisects $\overline{XY}$, and point C is not on $\overline{XY}$.



11. $\overline{XY}$ intersects $\overline{WV}$ at point A, so that $XA = VA$.



12. $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ are all in plane P and point X is the midpoint of each segment.



13. Yes, planes W and X intersect at $\overleftrightarrow{KL}$.

14. Yes, N, K, and M are collinear with L not on the line, so these points are coplanar.

15. No, Q is a point contained in plane W, M is a point contained in plane X, and J is a point on the intersection of the planes, so they are three noncollinear points.

16. No, $\overleftrightarrow{RP}$ and $\overleftrightarrow{MN}$ both intersect $\overleftrightarrow{JL}$ (which is contained in both planes) at two different points.

17. Yes, the line of intersection is contained in both planes.

18. No, there is not enough information given.

19. Yes, $\angle NKL$ and $\angle JKM$ are vertical angles.

20. Yes, the nonadjacent sides form a straight angle.

21. In order to determine that M is the midpoint of $\overline{AC}$ or $\overline{BD}$, the segments that would have to be marked as congruent are $\overline{AM}$ and $\overline{MC}$ or $\overline{DM}$ and $\overline{MB}$, respectively; Based on the diagram and markings, you can assume $\overline{AC}$ and $\overline{DB}$ intersect at point M, such that $\overline{AM} \cong \overline{MB}$ and $\overline{DM} \cong \overline{MC}$.

22. In order to assume that an angle measures 90°, the angle must be marked as such; Based on the diagram, you can assume two pairs of vertical angles, $\angle DMC$ and $\angle AMB$ or $\angle DMA$ and $\angle CMB$, and you can assume linear pairs, such as $\angle DMC$ and $\angle CMB$.

23. The statements that cannot be concluded are: C, D, F, and H.

24. one; Based on the Line-Point Postulate (Post. 2.2), line m contains at least two points. Because these two points are noncollinear with point C, based on the Three Point Postulate (Post. 2.4), there is exactly one plane that goes through line m and point C.

25. Two Point Postulate (Post. 2.1)

26. Line Intersection Postulate (Post. 2.3)

27. Two Point Postulate: Through any two points, there exists exactly one line.

- **a.** Conditional statement: If there are two points, then there exists exactly one line that passes through them.
- **b.** Converse: If there exists exactly one line that passes through a given point or points, then there are two points. (False)

  Inverse: If there are not two points, then there is not exactly one line that passes through them. (False)

  Contrapositive: If there is not exactly one line that passes through a given point or points, then there are not two points. (True)

28. Plane-Point Postulate: A plane contains at least three noncollinear points.

- **a.** Conditional statement: If a plane exists, then it contains at least three noncollinear points.

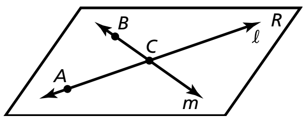
b. Converse: If a plane contains at least three noncollinear points, then the plane exists. (True)

Inverse: If no plane exists, then there are not three noncollinear points. (True)

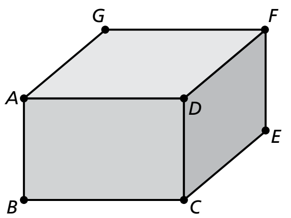
Contrapositive: If there are not three noncollinear points, then a plane has not been defined. (True)

29. Number of points to determine a line < number of points to determine a plane

30. yes; Let two lines ℓ and m intersect at point C. There must be a second point on each line, A in ℓ and B in m. Through the three noncollinear points A, B, and C, there exists exactly one plane R. Because A and C are in R, ℓ is in R. Because B and C are in R, m is in R.

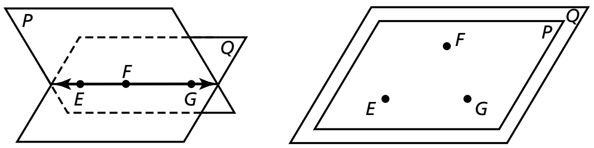


31. Yes, for example, three planes, $ABCD$, $DCEF$, and $DFGA$, have point D in common.



32. no; The postulate states that if two planes intersect, they will intersect in a line. But plans can be parallel and never intersect. For example, the ceiling and floor of a room are parallel.

33. Points E, F, and G must be collinear. They must be on the line that intersects plane P and plane Q; Points E, F, and G can be either collinear or not collinear.



34. *Sample answer:* The Line Intersection Postulate (Post. 2.3) would have to be altered. In spherical geometry, if two lines intersect, then their intersection is exactly two points. The two points of intersection would be the endpoints of a diameter.

Maintaining Mathematical Proficiency

35. Addition Property of Equality

$$t - 6 = -4$$
$$t - 6 + 6 = -4 + 6$$
$$t = 2$$

36. Division Property of Equality

$$3x = 21$$
$$\frac{3x}{3} = \frac{21}{3}$$
$$x = 7$$

37. Subtraction Property of Equality

$$9 + x = 13$$
$$9 - 9 + x = 13 - 9$$
$$x = 4$$

38. Multiplication Property of Equality

$$\frac{x}{7} = 5$$
$$7 \bullet \frac{x}{7} = 5 \bullet 7$$
$$x = 35$$

2.1–2.3 What Did You Learn? (p. 89)

1. "If you are in math class, then you are in geometry," is false. You could be in another math class. For example, you could be in Algebra I or Calculus.

"If you do your math homework, then you will do well on the test," is false. Some students can do all their homework. However, they may have test anxiety. In which case, they may not do well on the test.

"If it does not snow, then I will run outside" is false. On a day that it is not snowing you may feel too sick to run outside.

2. a. p: You go to the zoo to see a lion.

q: You will see a cat.

p	q	$q \to p$
T	T	T
T	F	F
F	T	T
F	F	T

b. p: You play a sport.

q: You wear a helmet.

p	q	$q \to p$
T	T	T
T	F	F
F	T	T
F	F	T

c. p: This month has 31 days.

q: It is not February.

p	q	$q \to p$
T	T	T
T	F	F
F	T	T
F	F	T

3. *Sample answer:* What about parallel lines? Do they intersect?