

# Key Review - Unit 10

Factor

$$\textcircled{1} \frac{10}{10a+30} \cdot \frac{a^2+9a-18}{a^2+10a+16} = \frac{\cancel{10}^1 \cdot (a+1)\cancel{(a+8)}}{\cancel{10}(a+3) \cdot (a+2)\cancel{(a+8)}}$$

Divide/cancel

$$= \boxed{\frac{a+1}{(a+3)(a+2)}}$$

$$\textcircled{2} \frac{\cancel{30}a \cancel{30}}{4} \cdot \frac{54}{\cancel{30} \cancel{30}} = \frac{54}{4} = \boxed{\frac{27}{2}}$$

$$\textcircled{3} \frac{5(\cancel{x+6})}{(\cancel{x+2})(x+5)} \cdot \frac{\cancel{(x+2)}}{\cancel{(x+6)}(x+9)} = \boxed{\frac{5}{(x+5)(x+9)}}$$

$$\textcircled{4} \frac{(v+6)(\cancel{v+9})}{(v-10)(\cancel{v+9})} \cdot \frac{\cancel{(v+9)}(v-9)}{(\cancel{v+9})(v+8)} = \boxed{\frac{(v+6)}{(v-10)}}$$

$$\textcircled{5} \frac{3(\cancel{m+1})}{(\cancel{m+9})(\cancel{m+1})} \cdot \frac{\cancel{m+4}}{(\cancel{m+9})(m+6)} = \boxed{\frac{3}{(m+9)(m+6)}}$$

$$\textcircled{6} \frac{(m+4)(\cancel{m+6})}{7(\cancel{m+4})} \cdot \frac{(m-8)\cancel{(m+5)}}{\cancel{(m+5)}(m+6)} = \boxed{\frac{m-8}{7}}$$

$$\textcircled{7} \frac{2}{7a(\cancel{a+8})} \cdot \frac{\cancel{(a+8)}(\cancel{a+5})}{9a^2(\cancel{a+8})} = \boxed{\frac{2}{63a^3}}$$

$$\textcircled{8} \frac{(\cancel{v+6})(v-5)}{\cancel{3v}(v+6)} \cdot \frac{2v}{9(v+4)} = \boxed{\frac{(v-5)}{9(v+4)}}$$

$$(10) \frac{\cancel{(x+5)}(x+4)}{\cancel{(x+4)}(x-2)} \cdot \frac{\cancel{(x-2)}(x-7)}{2(x+5)} = \boxed{\frac{(x-7)}{2}}$$

$$(11) \frac{5\cancel{x}(x+4)}{\cancel{(x+4)}} \cdot \frac{1\cancel{8}(7x-8)}{2\cancel{16}x^2(7x-8)} = \frac{5}{2(x+4)} = \boxed{\frac{5}{2}}$$

$$(12) \frac{3(3r+2)}{(r+8)(r+3)} \cdot \frac{\cancel{(r+3)}(r-2)}{3\cancel{(3+2)}} = \boxed{\frac{r-2}{r+8}}$$

$$(13) \frac{\cancel{(x+8)}(x-3)}{7(x-3)} \cdot \frac{(x-7)}{5(x+8)} = \boxed{\frac{x-7}{7}}$$

$$(14) \frac{\cancel{v-8}}{(v+9)(\cancel{v-8})} \cdot \frac{\cancel{(v+4)}(v-1)}{\cancel{(v+4)}(v+4)} = \boxed{\frac{v-1}{(v+9)(v+4)}}$$

$$(15) \frac{3(n+8)}{n+8} \cdot \frac{\cancel{(n-10)}(n+10)}{7(n-10)} = \boxed{\frac{3(n+10)}{7}}$$

Flip the 2<sup>nd</sup>

$$(16) \frac{\cancel{(n-6)}(n+4)}{\cancel{(n-6)}} \cdot \frac{n^2-2n-24}{n-6} \cdot \frac{n^2-5n-24}{n^2+n-72}$$

$$= \frac{\cancel{(n-6)}(n+4)}{n-6} \cdot \frac{\cancel{(n-8)}(n+3)}{(n+9)\cancel{(n+4)}} = \boxed{\frac{(n+4)(n+3)}{n+9}}$$

$$(17) \frac{9v^3-27v^2}{5v^3+10v^2} \div \frac{v^2+5v-24}{v^2+10v+16} \rightarrow \frac{9v^3-27v^2}{5v^3+10v^2} \cdot \frac{v^2+10v+16}{v^2+5v-24}$$

$$= \frac{9v^2(\cancel{v-3})}{5v^2(\cancel{v+2})} \cdot \frac{\cancel{(v+8)}(v+2)}{\cancel{(v+8)}(v-3)} = \boxed{\frac{9}{5}}$$

$$(18) \frac{3k-3}{6k-24} \div \frac{3k}{16k^2-64k} = \frac{3k-3}{6k-24} \cdot \frac{16k^2-64k}{3k}$$

$$= \frac{\cancel{3}(k-1)}{\cancel{2} \cancel{3}(k-4)} \cdot \frac{16k \cancel{(k-4)}}{3k} = \cancel{\frac{16k}{2}} \cdot \frac{8(k-1)}{42} = \boxed{\frac{8(k-1)}{2}}$$

$$(19) \frac{\cancel{n+7}}{8(n+2)} \cdot \frac{(n+5)(n+2)}{2(n+7)} = \boxed{\frac{n+5}{16}}$$

$$(20) \frac{\cancel{(b+8)}(b+6)}{\cancel{(b+8)}(b-9)} \cdot \frac{3(b-9)}{(b+6)} = \boxed{3}$$

$$(21) \frac{9(v-10)}{10v} \cdot \frac{\cancel{(v-10)} \cancel{(v+8)}}{\cancel{(v-10)} \cancel{(v+8)}} \cdot \frac{(v-6)(v+8)}{(v-10)(v+1)} = \boxed{\frac{9(v-6)}{10v}}$$

$$(22) \frac{(n+7)(n+2)}{n+7} \cdot \frac{(n+1)(n+9)}{3(n+7)} = \boxed{\frac{(n+2)(n+1)}{3}}$$

$$(23) \frac{2(x-1)}{x+5} \cdot \frac{\cancel{(x-7)}(x+5)}{\cancel{(x-7)}(x-9)} = \boxed{\frac{2(x-1)}{x-9}}$$

$$(24) \frac{4(n-8)}{n+10} \cdot \frac{(n+10)(n+5)}{(n-8)(n+5)} = \frac{4}{1} = \boxed{4}$$

$$(25) \frac{\cancel{(p-7)}(p-4)}{\cancel{(p-7)}(p+7)} \cdot \frac{(p-10)(p+7)}{1} = \frac{(p-4)(p-10)}{1} = \boxed{(p-4)(p-10)}$$

$$(26) \frac{8(\cancel{p-7})}{p+7} \cdot \frac{(p-4)(p+7)}{\cancel{(p-7)}(p-5)} = \boxed{\frac{8(p-4)}{p-5}}$$



$$(27) \frac{8(n+5)}{8(5n-7)} \cdot \frac{2n(5n-7)}{n+5} = \frac{2n}{1} = \boxed{2n}$$

$$(28) \frac{6p(5p-9)}{p+2} \cdot \frac{8(p+2)}{8(5p-9)} = \frac{6p}{1} = \boxed{6p}$$

$$(29) \frac{(x+10)(x+1)}{(x+10)} \cdot \frac{(x+10)(x+9)}{(x+10)(x-2)} = \boxed{\frac{(x+1)(x+9)}{x-2}}$$

$$(30) \frac{(x-8)(x+1)}{8(5x+3)} \cdot \frac{8(5x+3)}{(x-8)(x+8)} = \boxed{\frac{x+1}{x+8}}$$

sub  
trading

$$(31) \frac{5v}{4(v+1)} + \frac{-v+3}{4(v+1)} = \boxed{\frac{4v-3}{4v+4}}$$

same  
denominator

factor

$$(32) \frac{x-5}{2x^2-13x+16} + \frac{x+2}{2x^2-13x+16} = \boxed{\frac{3}{2x^2-13x+16}}$$

$$(33) \frac{2k-3}{9k(k+4)} + \frac{k+4}{9k(k+4)} = \boxed{\frac{3k+1}{9k^2+36k}}$$

$$(34) \frac{n+1}{n^2-n-2} + \frac{-n+1}{n^2-n-2} = \boxed{\frac{-2}{n^2-n-2}}$$

$$(35) \frac{3n+5}{3(n-5)} - \frac{n+5}{3(n-5)} = \frac{3n+5 - (n+5)}{3(n-5)} = \frac{3n+5 - n - 5}{3(n-5)} = \frac{2n}{3(n-5)}$$

(combine top)

$$\textcircled{35} \frac{\overset{(3n-5)}{12n}}{\underset{(3n-5)}{3} \cdot 6} - \frac{n+5}{3} = \frac{3n^2-5n}{3(3n-5)} + \frac{-3n+15}{3(3n-5)} = \boxed{\frac{3n^2-8n-15}{3(3n-5)}}$$

$$\textcircled{36} \frac{(x+1) 3x}{(x+1)(x+3)} - \frac{5(x+3)}{(x+1)(x+3)} = \frac{3x^2+3x}{(x+1)(x+3)} + \frac{-5x-15}{(x+1)(x+3)}$$

$$= \boxed{\frac{3x^2-2x-15}{(x+3)(x+1)}}$$

$$\textcircled{37} \frac{\overset{1}{2}}{\underset{2(x-4)}{2x-8}} + \frac{5x}{3} = \frac{(3)1}{(3)x-4} + \frac{5x(x-4)}{3(x-4)} = \frac{3}{3(x-4)} + \frac{5x^2-20x}{3(x-4)}$$

$$= \frac{5x^2-20x+3}{3(x-4)}$$

$$\textcircled{38} \frac{\overset{(2n+1)}{3}}{\underset{(2n+1)}{5n-2}} - \frac{6(5n-2)}{2n+1(5n-2)} = \frac{\overset{(6n+3)}{3}}{\underset{(2n+1)}{(5n-2)}} + \frac{\overset{(-30n+12)}{-6}}{\underset{(2n+1)}{(5n-2)}}$$

$$= \frac{-24n+15}{(5n-2)(2n+1)}$$

$$\textcircled{39} \frac{(x-6) 4x}{(x-6)(x-2)} + \frac{5(x-2)}{(x-6)(x-2)} = \frac{(4x^2-24x)}{(x-6)(x-2)} + \frac{5x-10}{(x-6)(x-2)} = \boxed{\frac{4x^2-19x-10}{(x-6)(x-2)}}$$

$$\textcircled{40} \frac{(6) 4x}{(6)x+3} + \frac{5x(x+3)}{6(x+3)} = \frac{24x}{6(x+3)} + \frac{5x^2+15x}{6(x+3)}$$

$$= \boxed{\frac{5x^2+39x}{6(x+3)}}$$

$$\textcircled{41} \frac{(a+1) 6a}{(a+1)5a+1} - \frac{4(5a+1)}{a+1(5a+1)} = \frac{6a^2+6a}{(a+1)(5a+1)} + \frac{-20a-4}{(a+1)(5a+1)} = \boxed{\frac{6a^2-14a-4}{(a+1)(5a+1)}}$$

$$(42) \quad \frac{(2) 2b}{(2) b+5} - \frac{3 (b+5)}{2 (b+5)} = \frac{4b}{2(b+5)} + \frac{-3b-15}{2(b+5)} = \frac{b-15}{2(b+5)}$$

$$(43) \quad \frac{(3) p-2}{(3) 2(2p+5)} + \frac{2 (2)(2p-5)}{3 (2)(2p+5)}$$
~~$$= \frac{3p-6}{6(2p+5)} + \frac{4(2p-5)}{6(2p+5)} = \frac{3p-6}{6(2p+5)} + \frac{8p-20}{6(2p+5)} = \frac{11p-26}{6(2p+5)} = \frac{11p-14}{6(2p+5)}$$~~

$$(44) \quad \frac{3}{(3) a-5} - \frac{2}{(3) a-5} = \frac{4}{3(a-5)} = \frac{6}{3(a-5)} - \frac{2a+10}{5(a-5)} = \frac{-2a+16}{3(a-5)}$$

$$(45) \quad \frac{(a-3)(a+6)}{(a-3)(a-4)} + \frac{2a(a-4)}{(a-3)(a-4)} = \frac{a^2+3a-18}{(a-3)(a-4)} + \frac{2a^2-8a}{(a-3)(a-4)}$$

$$= \frac{3a^2-5a-18}{(a-3)(a-4)}$$

$$(46) \quad \frac{2}{m+5} + \frac{2}{3m+6} - \frac{(m+2) 2}{(m+2)(m+5)} + \frac{2m(m+5)}{m+2(m+5)}$$

$$= \frac{2m+4}{(m+2)(m+5)} + \frac{2m^2+10m}{(m+2)(m+5)} = \frac{2m^2+12m+4}{(m+2)(m+5)}$$

$$(47) \quad \frac{(n-3) 2}{(n-3) 3} - \frac{6 (3)}{n-3(3)} = \frac{2n-6}{3(n-3)} + \frac{-18}{3(n-3)} = \frac{2n-24}{3(n-3)}$$

$$(48) \quad \frac{(a-4) 6a}{(a-4) a-2} - \frac{5 (a-2)}{a-4 (a-2)} = \frac{6a^2-24a}{(a-4)(a-2)} + \frac{-5a+10}{(a-4)(a-2)}$$

$$= \frac{6a^2-29a+10}{(a-4)(a-2)}$$

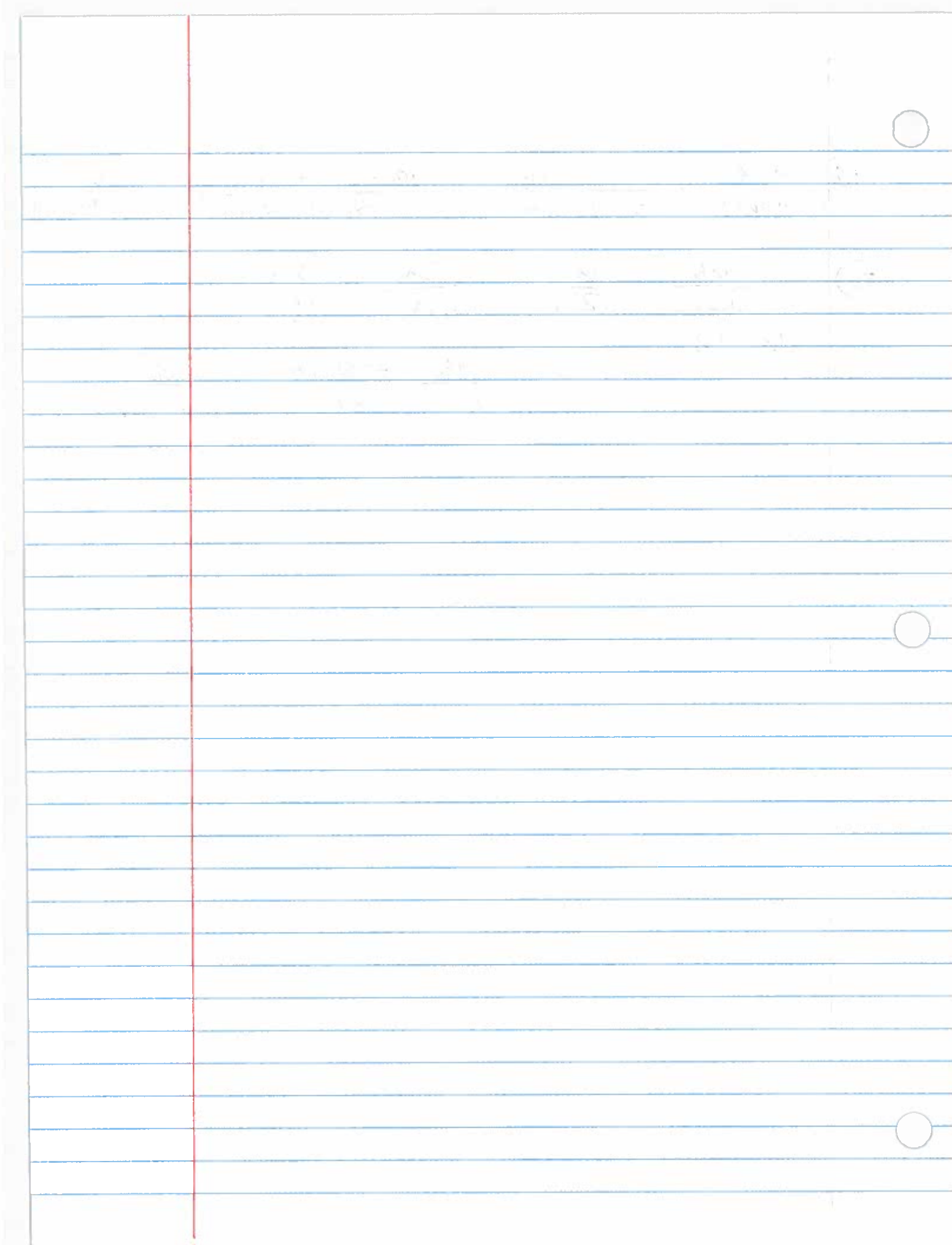


$$\textcircled{49} \frac{5v-4}{(5v-4)} \frac{2}{5v+4} + \frac{2}{5v-4} \frac{5v+4}{(5v+4)} = \frac{\textcircled{10v-8}}{(5v-4)(5v+4)} + \frac{\textcircled{10v+8}}{(5v-4)(5v+4)} = \boxed{\frac{20v}{(5v-4)(5v+4)}}$$

$$\textcircled{50} \frac{\cancel{3}b}{\cancel{2b+6}} - \frac{\cancel{6}3}{\cancel{2}1} = \frac{3b}{b+3} - \frac{3(b+3)}{1(b+3)}$$

~~1(b+3)~~

$$\frac{\cancel{3b}}{b+3} + \frac{-\cancel{3b}-9}{b+3} = \boxed{\frac{-9}{b+3}}$$





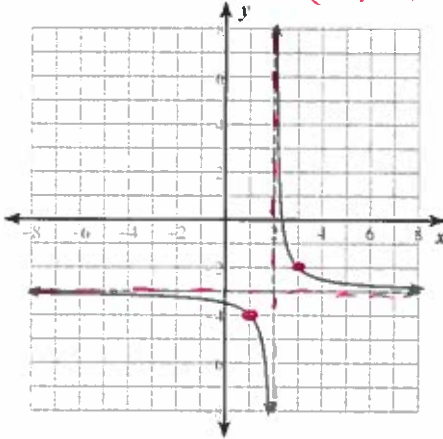
Identify the points of discontinuity and vertical asymptotes of each. Then sketch the graph.

51)  $f(x) = \frac{1}{x-2} - 3$

$(2, -3)$   $\frac{1}{1}$  and  $\frac{1}{1}$

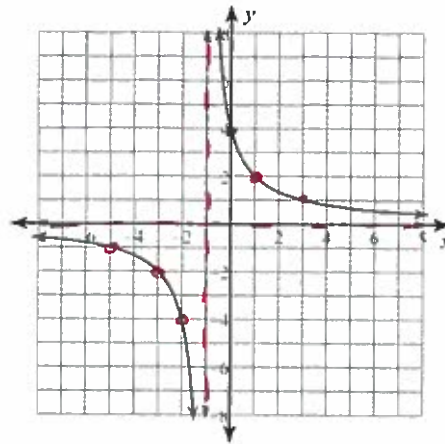
Discontinuities 2  
Vertical Asym.  $x = 2$

$\sqrt{1} = 1$



52)  $f(x) = \frac{4}{x+1} + 0$   $(-1, 0)$

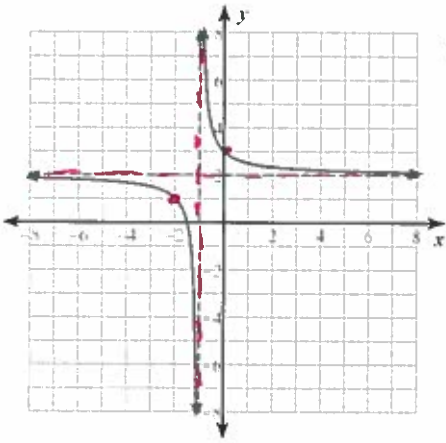
Discontinuities -1  
Vertical Asym.  $x = -1$



53)  $f(x) = \frac{1}{x+1} + 2$

$(-1, 2)$

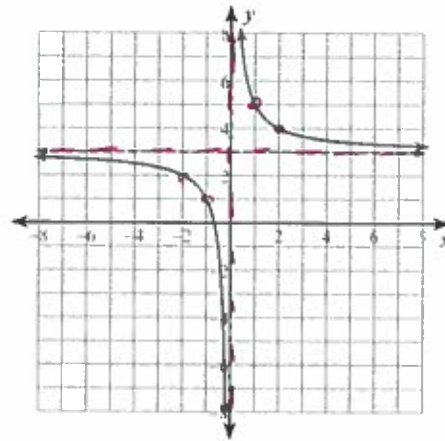
Discontinuities -1  
Vertical Asym.  $x = -1$



54)  $f(x) = \frac{2}{x} + 3$

$(0, 3)$

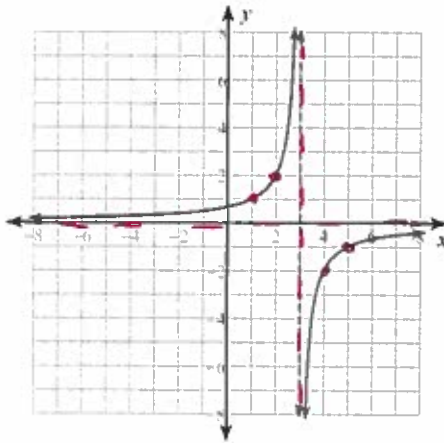
Discontinuities 0  
Vertical Asym.  $x = 0$





$$55) f(x) = \frac{2}{x-3}$$

$$-\frac{2}{1} \text{ and } -\frac{1}{2}$$

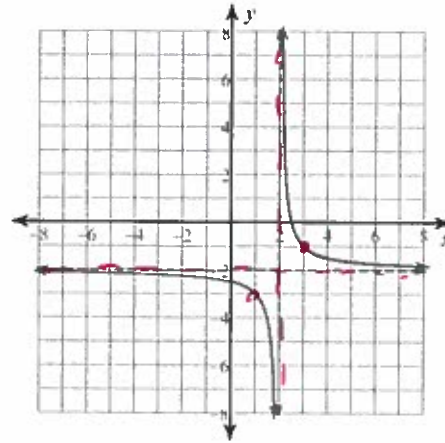


Discontinuities: 3  
Vertical Asym.  $x=3$

$$(3,0)$$

$$56) f(x) = \frac{1}{x-2} - 2$$

$$(2,-2)$$

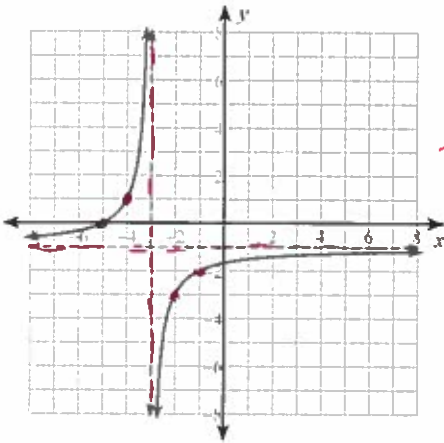


Discontinuities: 2  
Vertical Asym.  $x=2$

$$\frac{1}{1} \quad \frac{1}{1}$$

$$\sqrt{1}$$

$$57) f(x) = \frac{2}{x+3} - 1$$



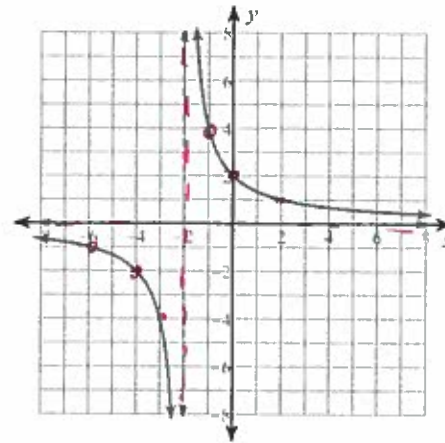
Discontinuities: -3  
Vertical Asym.  $x=-3$

$$-\frac{2}{1} \quad -\frac{1}{2}$$

$$\sqrt{2}$$

$$58) f(x) = \frac{4}{x+2}$$

$$(-2,0)$$



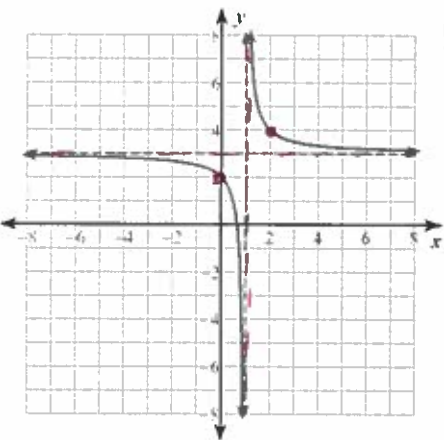
Discontinuities: +2  
Vertical Asym.  $x=-2$

$$\frac{4}{1} \quad \frac{1}{4}$$

$$\sqrt{4} = 2 \checkmark$$

$$59) f(x) = \frac{1}{x-1} + 3$$

$$(1,3)$$



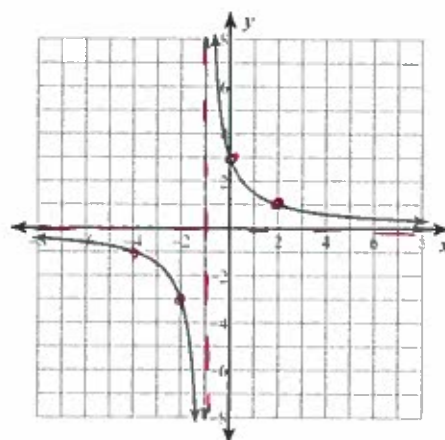
Discontinuities: 1  
Vertical Asym.  $x=1$

$$\frac{1}{1} \quad \frac{1}{1}$$

$$\sqrt{1}$$

$$60) f(x) = \frac{3}{x+1}$$

$$(-1,0)$$



Discontinuities: -1  
Vertical Asym.  $x=-1$

$$\frac{3}{1} \quad \frac{1}{3} \quad \sqrt{3}$$





# Graphing Rational Functions with Holes

For each function, find the holes, vertical asymptotes, horizontal asymptotes, and domain. Use your calculator or Desmos to help you graph the equation.

EATS  
NAME

$$\frac{x^2 + 7x + 6}{x^2 - 4x - 5} = \frac{(x+6)(x+1)}{(x+1)(x-5)}$$

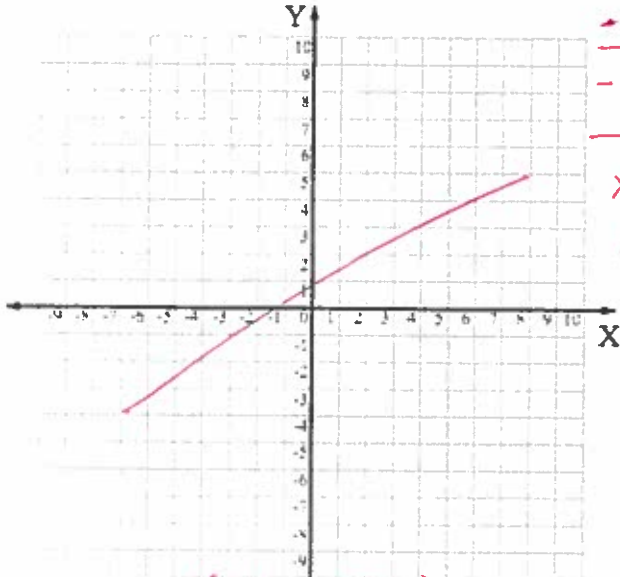
$$\frac{1}{1} = 1$$

Hole:

$$\frac{-1+6}{-1-5} = \frac{5}{-4}$$

$$x-5=0$$

$$x=5$$



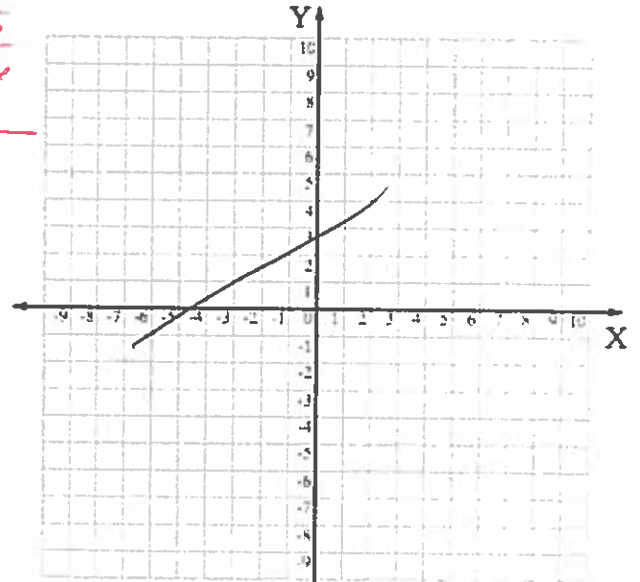
Holes:  $(-1, -5/4)$

VA:  $x=5$

HA:  $y=1$

Domain:  $(-\infty, -1) \cup (-1, 5) \cup (5, \infty)$

$$\frac{x^2 + 4x - 12}{x^2 - 5x + 6} = \frac{(x+6)(x-2)}{(x-2)(x+1)}$$



Holes: None

VA:  $x=6, x=-1$

HA:  $y=1$

Domain:  $(-\infty, -1) \cup (-1, 6) \cup (6, \infty)$

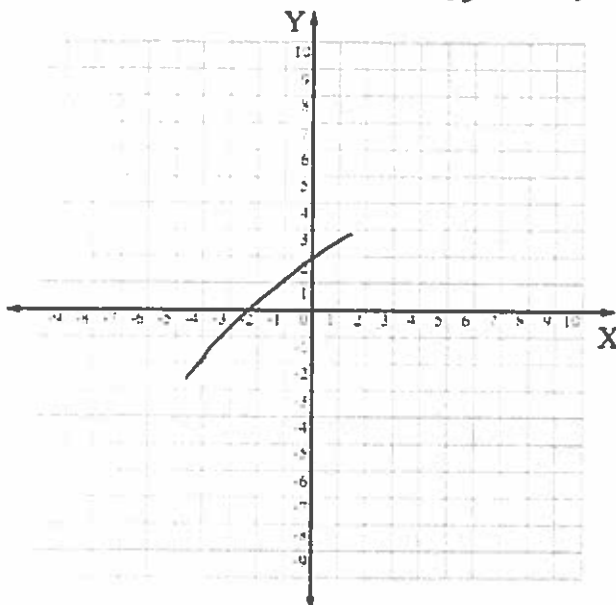
BOBO

$$\frac{2x+6}{x^2+7x+12} = \frac{2(x+3)}{(x+3)(x+4)}$$

$$\frac{2}{-3+4} = \frac{2}{1} = 2$$

$$x+4=0$$

$$x=-4$$



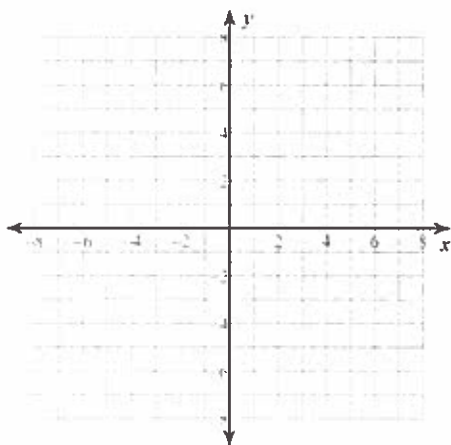
Holes:  $(-3, 2)$

VA:  $x=-4$

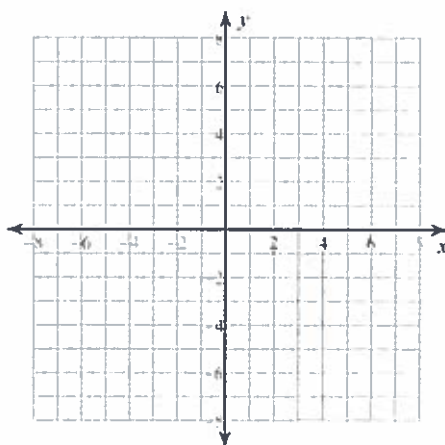
HA:  $y=0$

Domain:  $(-\infty, -4) \cup (-4, -3) \cup (-3, \infty)$

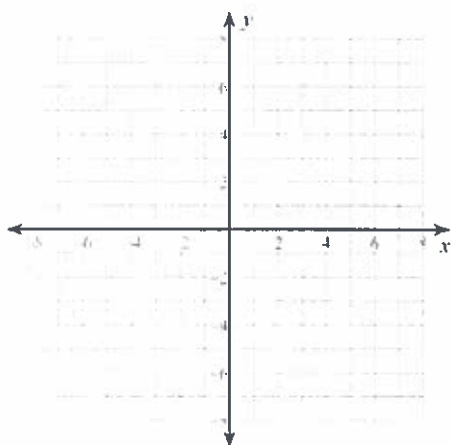
$$55) f(x) = -\frac{2}{x-3}$$



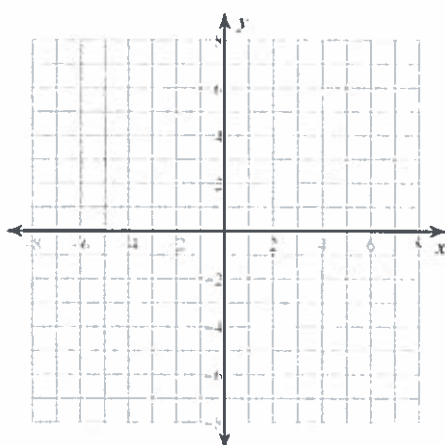
$$56) f(x) = \frac{1}{x-2} - 2$$



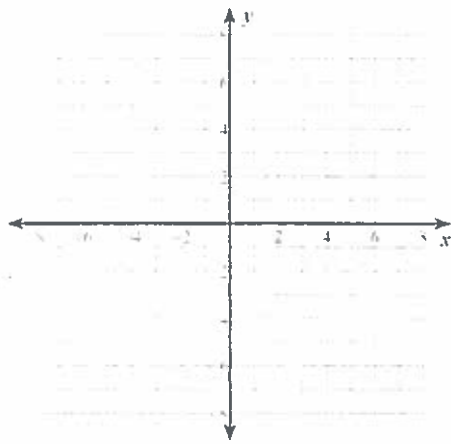
$$57) f(x) = -\frac{2}{x+3} - 1$$



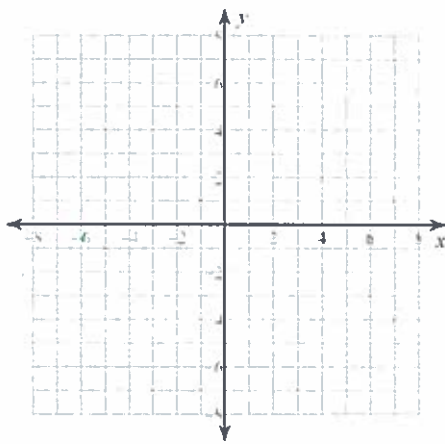
$$58) f(x) = \frac{4}{x+2}$$



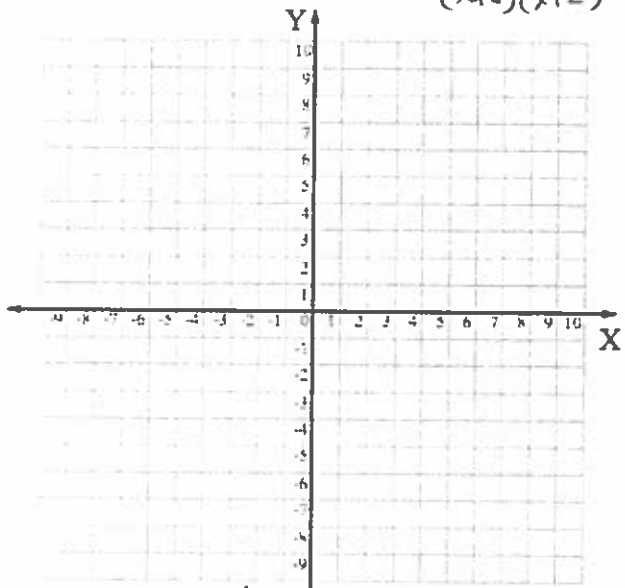
$$59) f(x) = \frac{1}{x-1} + 3$$



$$60) f(x) = \frac{3}{x+1}$$



EATS  $\frac{1}{1} \frac{x^2 - 7x + 12}{x^2 + 8x + 12} = \frac{(x-3)(x-4)}{(x+6)(x+2)}$



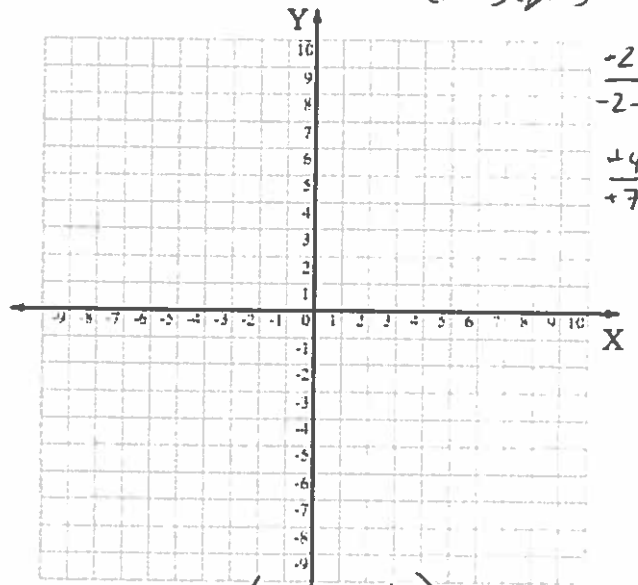
Holes: None

VA:  $x = -6, x = -2$

HA:  $y = 1$

Domain:  $(-\infty, -6) \cup (-6, -2) \cup (-2, \infty)$

EATS  $\frac{x^2 - 4}{x^2 - 3x - 10} = \frac{(x-2)(x+2)}{(x-5)(x+2)}$



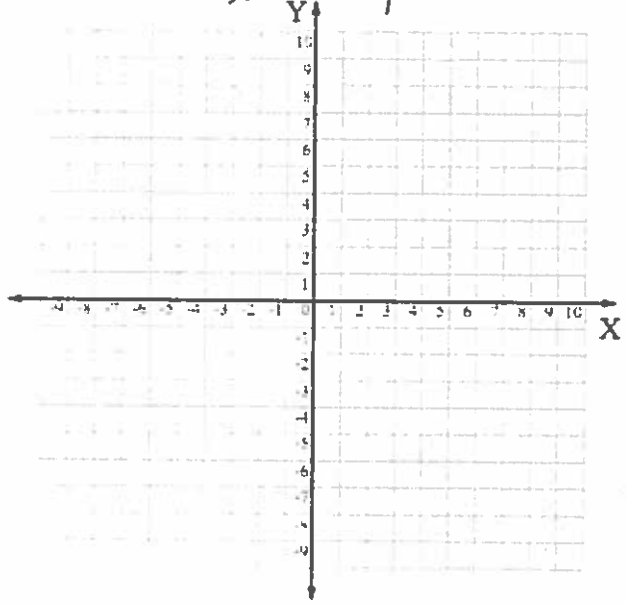
Holes:  $(-2, 4/7)$

VA:  $x = 5$

HA:  $y = 1$

Domain:  $(-\infty, -2) \cup (-2, 5) \cup (5, \infty)$

$\frac{1}{2} \frac{-6}{-5} \leftarrow \frac{2x^2 - 5x - 3}{x^2 - 4x + 4} = \frac{(2x-3)(2x+1)}{(x-2)(x-2)}$



Holes: None

VA:  $x = 2$

HA:  $y = 2$

Domain:  $(-\infty, 2) \cup (2, \infty)$

EATS  $\frac{2}{1}$

