

Overview of Section 6.2

Introduction

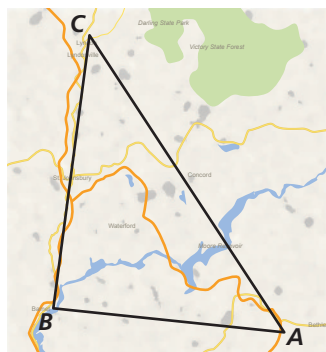
- In the last lesson, perpendicular bisectors and angle bisectors were reviewed and their properties explored.
- In this lesson, the three perpendicular bisectors in a triangle are constructed and the circumcenter, the *point of concurrency*, is found. The *circumcenter* is the point that is equidistant from all three vertices of the triangle.
- In the second part of the lesson, the three angle bisectors in a triangle are constructed and the incenter, the *point of concurrency*, is found. The *incenter* is the point that is equidistant from all three sides of the triangle.

Resources

- Tracing paper is helpful when the explorations are done by paper folding.

Teaching Strategy

- For either of the constructions in this lesson (circumcenter and incenter), consider using a local map that your students would be familiar with. Take a screen shot of the image and import it to your dynamic geometry software.
- In the example shown, I currently live at point *A*, I lived for many years at point *B*, and I work at point *C*.
- Think of the type of applications for the circumcenter and incenter, and select an image that will make sense.



Pacing Suggestion

- The explorations allow students to explore and discover two relationships that will be stated as theorems in the formal lesson. The constructions and investigations are not long and should be followed with the statement of the Circumcenter Theorem (Thm. 6.5) in the lesson. You may choose to omit the two compass constructions in the lesson.

G.5.C The student is expected to use the constructions of congruent segments, congruent angles, angle bisectors, and perpendicular bisectors to make conjectures about geometric relationships.

G.6.D The student is expected to verify theorems about the relationships in triangles, including proof of the Pythagorean Theorem, the sum of interior angles, base angles of isosceles triangles, midsegments, and medians, and apply these relationships to solve problems.

Laurie's Notes

Exploration

Motivate

- There are three high schools located in different parts of the city. You want to open an after-school tutoring service for students from all of the schools. Where would you locate your service so that any student has the same distance to drive?
- Let students discuss the scenario.
- Explain to students that after this lesson they should be able to give an expert opinion about the location of the tutoring service.

Exploration 1

- This is a quick construction using dynamic geometric software. It is not necessary for the coordinate axes to be visible.
- Students observe that the three perpendicular bisectors appear to intersect at one point.
- **Turn and Talk:** "How could you show that the lines intersect at just one point and not at points that are simply very close together?" One way is to label the points of intersection of each pair of lines D , E , and F . If coordinates appear, students will note that D , E , and F are the same point. If the construction was not done in a coordinate plane, measure \overline{DE} , \overline{DF} , and \overline{FE} . All segments will have a length of 0, meaning D , E , and F are the same point.
- **TEKS G.1.D, G.1.F, and G.1.G:** Draw the circle. "What conjecture(s) can you make?" [The circle goes through the three vertices of the triangle.](#)
- **TEKS G.1.C:** Students can quickly explore other triangles by clicking and dragging on any vertex of the triangle.
- Students may make other observations, for instance that point D is the same distance from all three vertices because D is the center of the circle.

Exploration 2

- This also is a quick construction.
- Students observe that the three angle bisectors appear to intersect at one point. Have students confirm that there is but one point and not three points that are simply close together.
- **TEKS G.1.D, G.1.F, and G.1.G:** Draw the circle. "What conjecture(s) can you make?" [The circle touches each side of the triangle.](#)
- **TEKS G.1.C:** Students can quickly explore other triangles by clicking and dragging on any vertex of the triangle.
- Students may make other observations—for instance, that the circle stays inside the sides of the triangle.

Communicate Your Answer

- Students should state that the three perpendicular bisectors of the sides of a triangle intersect at one point, and the three angle bisectors of a triangle intersect at one point.

Connecting to Next Step

- The explorations are related to the theorems presented in the lesson, helping students make sense of the theorems before proving them to be true.

6.2 Bisectors of Triangles



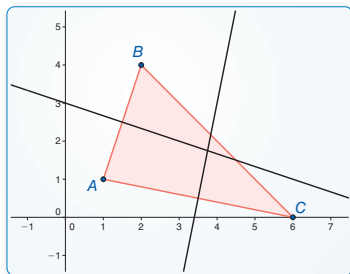
TEXAS ESSENTIAL
KNOWLEDGE AND SKILLS
G.5.C
G.6.D

Essential Question What conjectures can you make about the perpendicular bisectors and the angle bisectors of a triangle?

EXPLORATION 1 Properties of the Perpendicular Bisectors of a Triangle

Work with a partner. Use dynamic geometry software. Draw any $\triangle ABC$.

- Construct the perpendicular bisectors of all three sides of $\triangle ABC$. Then drag the vertices to change $\triangle ABC$. What do you notice about the perpendicular bisectors?
- Label a point D at the intersection of the perpendicular bisectors.
- Draw the circle with center D through vertex A of $\triangle ABC$. Then drag the vertices to change $\triangle ABC$. What do you notice?



Sample

Points

$$A(1, 1)$$

$$B(2, 4)$$

$$C(6, 0)$$

Segments

$$BC = 5.66$$

$$AC = 5.10$$

$$AB = 3.16$$

Lines

$$x + 3y = 9$$

$$-5x + y = -17$$

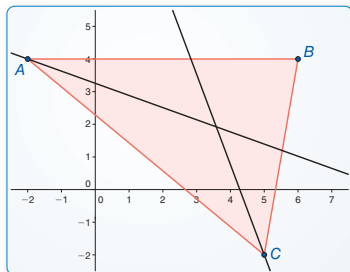
ANALYZING MATHEMATICAL RELATIONSHIPS

To be proficient in math, you need to see complicated things as single objects or as being composed of several objects.

EXPLORATION 2 Properties of the Angle Bisectors of a Triangle

Work with a partner. Use dynamic geometry software. Draw any $\triangle ABC$.

- Construct the angle bisectors of all three angles of $\triangle ABC$. Then drag the vertices to change $\triangle ABC$. What do you notice about the angle bisectors?
- Label a point D at the intersection of the angle bisectors.
- Find the distance between D and \overline{AB} . Draw the circle with center D and this distance as a radius. Then drag the vertices to change $\triangle ABC$. What do you notice?



Sample

Points

$$A(-2, 4)$$

$$B(6, 4)$$

$$C(5, -2)$$

Segments

$$BC = 6.08$$

$$AC = 9.22$$

$$AB = 8$$

Lines

$$0.35x + 0.94y = 3.06$$

$$-0.94x - 0.34y = -4.02$$

Communicate Your Answer

- What conjectures can you make about the perpendicular bisectors and the angle bisectors of a triangle?

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ANSWERS

1–2. See Additional Answers.

- The perpendicular bisectors of the sides of a triangle meet at a point that is the same distance from each vertex of the triangle. The angle bisectors of a triangle meet at a point that is the same distance from each side of the triangle.

SUPPORTING English Language Learners

Read aloud the Essential Question. Have students work in pairs of mixed language abilities to complete the explorations.

Beginning/Advanced Beginning students read aloud the headings and Advanced students read the instructions, providing synonymous phrases when needed to improve understanding. Then they work together to complete the explorations.

Intermediate/Advanced High Intermediate students read aloud the headings and instructions, while Advanced High students provide synonymous phrases to improve understanding. Then they work together to complete the explorations.

ELPS 3.B.3 Expand and internalize initial English vocabulary by learning and using routine language needed for classroom communication.

Notebook Development

Have students record the Circumcenter Theorem (Thm. 6.5) and the Incenter Theorem (Thm. 6.6) from this section in their notebooks. For each theorem, include a sketch of the point of concurrency and an example of how to use the theorem to find unknown measures.

6.2 Lesson

Core Vocabulary

concurrent, p. 314
 point of concurrency, p. 314
 circumcenter, p. 314
 incenter, p. 317

Previous

perpendicular bisector
 angle bisector

What You Will Learn

- ▶ Use and find the circumcenter of a triangle.
- ▶ Use and find the incenter of a triangle.

Using the Circumcenter of a Triangle

When three or more lines, rays, or segments intersect in the same point, they are called **concurrent** lines, rays, or segments. The point of intersection of the lines, rays, or segments is called the **point of concurrency**.

In a triangle, the three perpendicular bisectors are concurrent. The point of concurrency is the **circumcenter** of the triangle.

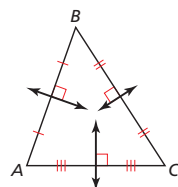
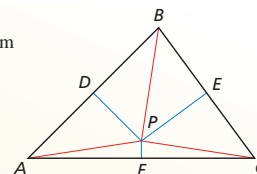
Theorems

Theorem 6.5 Circumcenter Theorem

The circumcenter of a triangle is equidistant from the vertices of the triangle.

If \overline{PD} , \overline{PE} , and \overline{PF} are perpendicular bisectors, then $PA = PB = PC$.

Proof p. 314



PROOF Circumcenter Theorem

Given $\triangle ABC$; the perpendicular bisectors of \overline{AB} , \overline{BC} , and \overline{AC}

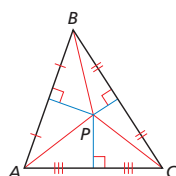
Prove The perpendicular bisectors intersect in a point; that point is equidistant from A, B, and C.

Plan for Proof Show that P, the point of intersection of the perpendicular bisectors of \overline{AB} and \overline{BC} , also lies on the perpendicular bisector of \overline{AC} . Then show that point P is equidistant from the vertices of the triangle.

Plan in Action	STATEMENTS	REASONS
	1. $\triangle ABC$; the perpendicular bisectors of \overline{AB} , \overline{BC} , and \overline{AC}	1. Given
	2. The perpendicular bisectors of \overline{AB} and \overline{BC} intersect at some point P.	2. Because the sides of a triangle cannot be parallel, these perpendicular bisectors must intersect in some point. Call it P.
	3. Draw \overline{PA} , \overline{PB} , and \overline{PC} .	3. Two Point Postulate (Post. 2.1)
	4. $PA = PB$, $PB = PC$	4. Perpendicular Bisector Theorem (Thm. 6.1)
	5. $PA = PC$	5. Transitive Property of Equality
	6. P is on the perpendicular bisector of \overline{AC} .	6. Converse of the Perpendicular Bisector Theorem (Thm. 6.2)
	7. $PA = PB = PC$. So, P is equidistant from the vertices of the triangle.	7. From the results of Steps 4 and 5 and the definition of equidistant

STUDY TIP

Use diagrams like the one below to help visualize your proof.



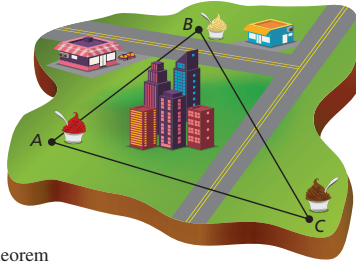
Laurie's Notes Teacher Actions

- Ask what the word *concurrent* means. Relate *point of concurrency* with the two explorations.
- ? Draw $\triangle ABC$ and sketch one perpendicular bisector. "What do you know about any point on this perpendicular bisector?" *It is equidistant from the two vertices of the bisected side.*
- ? Repeat for a second perpendicular bisector. Where the two bisectors intersect ask, "What is true about this point?" *It is equidistant from all three vertices of the triangle.*
- State the Circumcenter Theorem and discuss the proof.

EXAMPLE 1 Solving a Real-Life Problem

Three snack carts sell frozen yogurt from points A , B , and C outside a city. Each of the three carts is the same distance from the frozen yogurt distributor.

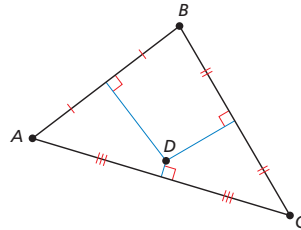
Find the location of the distributor.



SOLUTION

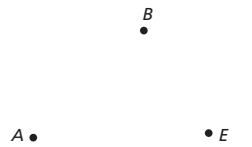
The distributor is equidistant from the three snack carts. The Circumcenter Theorem shows that you can find a point equidistant from three points by using the perpendicular bisectors of the triangle formed by those points.

Copy the positions of points A , B , and C and connect the points to draw $\triangle ABC$. Then use a ruler and protractor to draw the three perpendicular bisectors of $\triangle ABC$. The circumcenter D is the location of the distributor.



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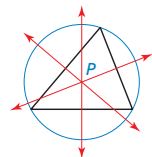
- Three snack carts sell hot pretzels from points A , B , and E . What is the location of the pretzel distributor if it is equidistant from the three carts? Sketch the triangle and show the location.



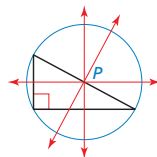
READING

The prefix *circum-* means "around" or "about," as in *circumference* (distance around a circle).

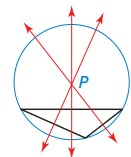
The circumcenter P is equidistant from the three vertices, so P is the center of a circle that passes through all three vertices. As shown below, the location of P depends on the type of triangle. The circle with center P is said to be *circumscribed* about the triangle.



Acute triangle
 P is inside triangle.



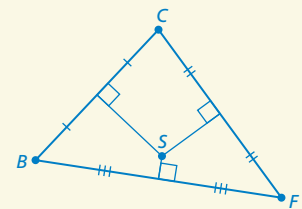
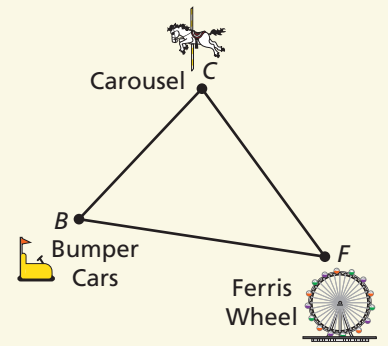
Right triangle
 P is on triangle.



Obtuse triangle
 P is outside triangle.

Extra Example 1

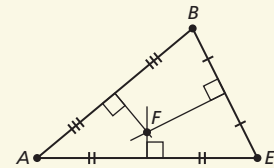
A carnival operator wants to locate a food stand so that it is the same distance from the carousel (C), the Ferris wheel (F), and the bumper cars (B). Find the location of the food stand.



The circumcenter S of $\triangle BCF$ is the location of the food stand.

MONITORING PROGRESS ANSWER

- The pretzel distributor is located at point F , which is the circumcenter of $\triangle ABE$.



Laurie's Notes **Teacher Actions**

- Extension:** To make Example 1 more interesting, print a street map of your area. Locate three points that are known to students and describe a plausible context. Locate the circumcenter for the triangle formed by the three fixed points.
- Fact-First Questioning:** "The circumcenter could be located on the side of a triangle. What type of triangle would it be?" *If students explored the circumcenter with dynamic geometry software, they likely know the triangle would be a right triangle.* Note the summary at the bottom of the page.

Extra Example 2

Find the coordinates of the circumcenter of $\triangle DEF$ with vertices $D(6, 4)$, $E(-2, 4)$, and $F(-2, -2)$. (2, 1)

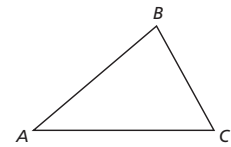
MONITORING PROGRESS

ANSWERS

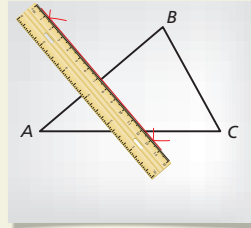
- (-4, 2)
- (0, -1)

CONSTRUCTION Circumscribing a Circle About a Triangle

Use a compass and straightedge to construct a circle that is circumscribed about $\triangle ABC$.

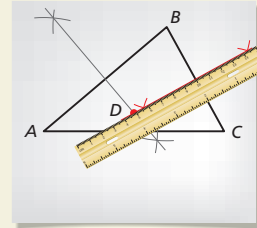


Step 1



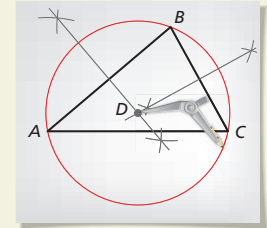
Draw a bisector Draw the perpendicular bisector of \overline{AB} .

Step 2



Draw a bisector Draw the perpendicular bisector of \overline{BC} . Label the intersection of the bisectors D . This is the circumcenter.

Step 3



Draw a circle Place the compass at D . Set the width by using any vertex of the triangle. This is the radius of the *circumcircle*. Draw the circle. It should pass through all three vertices A , B , and C .

STUDY TIP

Note that you only need to find the equations for two perpendicular bisectors. You can use the perpendicular bisector of the third side to verify your result.

EXAMPLE 2 Finding the Circumcenter of a Triangle

Find the coordinates of the circumcenter of $\triangle ABC$ with vertices $A(0, 3)$, $B(0, -1)$, and $C(6, -1)$.

SOLUTION

Step 1 Graph $\triangle ABC$.

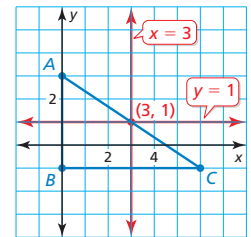
Step 2 Find equations for two perpendicular bisectors. Use the Slopes of Perpendicular Lines Theorem (Theorem 3.14), which states that horizontal lines are perpendicular to vertical lines.

The midpoint of \overline{AB} is $(0, 1)$. The line through $(0, 1)$ that is perpendicular to \overline{AB} is $y = 1$.

The midpoint of \overline{BC} is $(3, -1)$. The line through $(3, -1)$ that is perpendicular to \overline{BC} is $x = 3$.

Step 3 Find the point where $x = 3$ and $y = 1$ intersect. They intersect at $(3, 1)$.

► So, the coordinates of the circumcenter are $(3, 1)$.



USING PROBLEM-SOLVING STRATEGIES

Because $\triangle ABC$ is a right triangle, the circumcenter lies on the triangle.

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Find the coordinates of the circumcenter of the triangle with the given vertices.

- $R(-2, 5)$, $S(-6, 5)$, $T(-2, -1)$
- $W(-1, 4)$, $X(1, 4)$, $Y(1, -6)$

Laurie's Notes Teacher Actions

- Construct the circumcenter. Vary the type of triangle (right, obtuse, acute) used by students.
- You might choose to modify Example 2 so that the perpendicular bisectors are not vertical or horizontal. Students should have the prerequisite skills to find the point of intersection for a more challenging problem. Alternately, consider using dynamic geometry software with a map that has been imported. See the *Teaching Strategy* on page T-312.

Using the Incenter of a Triangle

Just as a triangle has three perpendicular bisectors, it also has three angle bisectors. The angle bisectors of a triangle are also concurrent. This point of concurrency is the **incenter** of the triangle. For any triangle, the incenter always lies inside the triangle.

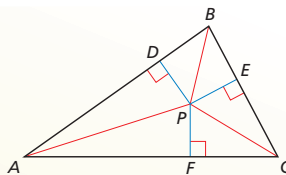
Theorem

Theorem 6.6 Incenter Theorem

The incenter of a triangle is equidistant from the sides of the triangle.

If \overline{AP} , \overline{BP} , and \overline{CP} are angle bisectors of $\triangle ABC$, then $PD = PE = PF$.

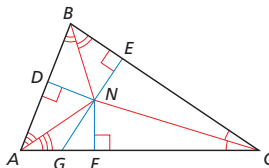
Proof Ex. 38, p. 321



EXAMPLE 3 Using the Incenter of a Triangle

In the figure shown, $ND = 5x - 1$ and $NE = 2x + 11$.

- Find NF .
- Can NG be equal to 18? Explain your reasoning.



SOLUTION

- N is the incenter of $\triangle ABC$ because it is the point of concurrency of the three angle bisectors. So, by the Incenter Theorem, $ND = NE = NF$.

Step 1 Solve for x .

$$ND = NE \quad \text{Incenter Theorem}$$

$$5x - 1 = 2x + 11 \quad \text{Substitute.}$$

$$x = 4 \quad \text{Solve for } x.$$

Step 2 Find ND (or NE).

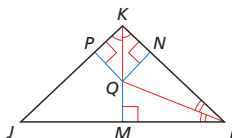
$$ND = 5x - 1 = 5(4) - 1 = 19$$

► So, because $ND = NF$, $NF = 19$.

- Recall that the shortest distance between a point and a line is a perpendicular segment. In this case, the perpendicular segment is \overline{NF} , which has a length of 19. Because $18 < 19$, NG cannot be equal to 18.

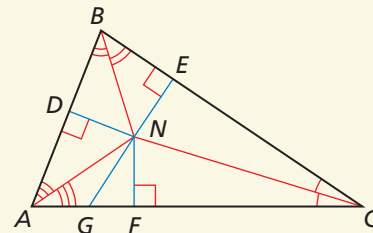
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- In the figure shown, $QM = 3x + 8$ and $QN = 7x + 2$. Find QP .



Extra Example 3

In the figure shown, $NE = 6x + 1$ and $NF = 4x + 15$.



- Find ND . 43
- Can $NB = 40$? Explain your reasoning.

No. The shortest distance from a point to a line is the length of the perpendicular segment from the point to the line. In this case, the perpendicular segment is \overline{ND} , which has a length of 43. Because $40 < 43$, NB cannot equal 40.

MONITORING PROGRESS

ANSWER

- 12.5

Laurie's Notes Teacher Actions

? **TEKS G.1.D, G.1.F, and G.1.G and Fact-First Questioning:** "The incenter of a triangle is always in the interior of the triangle. Explain why." **Listen for correct reasoning involving the three angle bisectors intersecting at a point in the interior of all three angles.**

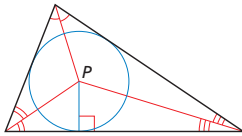
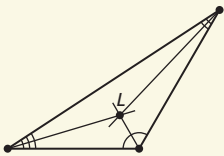
- **Extension:** To make Example 3 more interesting, print a street map of your area. Locate three points that are known to students and describe a plausible context. Locate the incenter for the triangle formed by the three fixed points.

Extra Example 4

A school has fenced in an area in the shape of a scalene triangle to use for a new playground. The school wants to place a swing set where it will be the same distance from all three fences. Should the swing set be placed at the *circumcenter* or the *incenter* of the triangular playground? Explain. **The incenter of a triangle is equidistant from the sides of the triangle, so the swing set should be at the incenter.**

MONITORING PROGRESS ANSWER

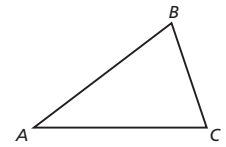
5.



Because the incenter P is equidistant from the three sides of the triangle, a circle drawn using P as the center and the distance to one side of the triangle as the radius will just touch the other two sides of the triangle. The circle is said to be *inscribed* within the triangle.

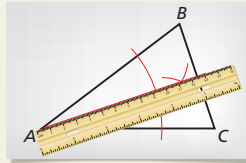
CONSTRUCTION Inscribing a Circle Within a Triangle

Use a compass and straightedge to construct a circle that is inscribed within $\triangle ABC$.



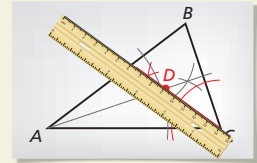
SOLUTION

Step 1



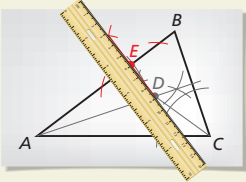
Draw a bisector Draw the angle bisector of $\angle A$.

Step 2



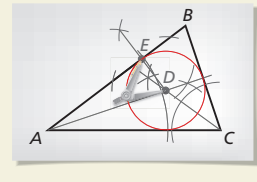
Draw a bisector Draw the angle bisector of $\angle C$. Label the intersection of the bisectors D . This is the incenter.

Step 3



Draw a perpendicular line Draw the perpendicular line from D to AB . Label the point where it intersects AB as E .

Step 4



Draw a circle Place the compass at D . Set the width to E . This is the radius of the *incircle*. Draw the circle. It should touch each side of the triangle.

EXAMPLE 4 Solving a Real-Life Problem

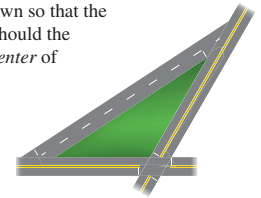
USING PRECISE MATHEMATICAL LANGUAGE

Pay close attention to how a problem is stated. The city wants the lamppost to be the *same distance* from the three streets, not from where the streets intersect.

A city wants to place a lamppost on the boulevard shown so that the lamppost is the same distance from all three streets. Should the location of the lamppost be at the *circumcenter* or *incenter* of the triangular boulevard? Explain.

SOLUTION

Because the shape of the boulevard is an obtuse triangle, its circumcenter lies outside the triangle. So, the location of the lamppost cannot be at the circumcenter. The city wants the lamppost to be the same distance from all three streets. By the Incenter Theorem, the incenter of a triangle is equidistant from the sides of a triangle.



► So, the location of the lamppost should be at the incenter of the boulevard.

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5. Draw a sketch to show the location L of the lamppost in Example 4.

Laurie's Notes Teacher Actions

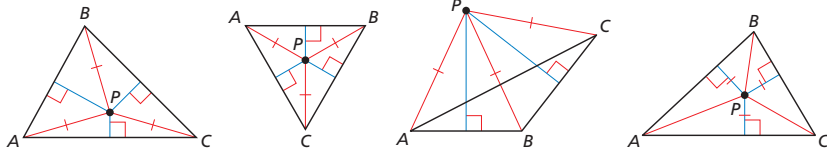
- Construct the incenter. Vary the type of triangle (right, obtuse, acute) used by students.
- **TEKS G.1.G:** Note that Example 4 states that the lamppost is to be the same distance from the three streets, not from where they intersect.
- ? "Can you think of other contexts where knowing the incenter would be necessary?" *Answers will vary.*

Closure

- **Exit Ticket:** Explain what the circumcenter and incenter of a triangle are and how they can be found. **The circumcenter of a triangle is equidistant from the vertices of the triangle. To find the circumcenter, find the intersection of the perpendicular bisectors of the sides of the triangle. The incenter of a triangle is equidistant from the sides of the triangle. To find the incenter, find the intersection of the angle bisectors of the interior angles of the triangle.**

Vocabulary and Core Concept Check

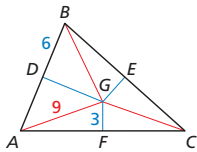
- VOCABULARY** When three or more lines, rays, or segments intersect in the same point, they are called _____ lines, rays, or segments.
- WHICH ONE DOESN'T BELONG?** Which triangle does *not* belong with the other three? Explain your reasoning.



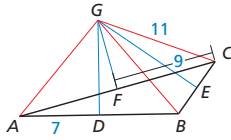
Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, the perpendicular bisectors of $\triangle ABC$ intersect at point G and are shown in blue. Find the indicated measure.

3. Find BG .

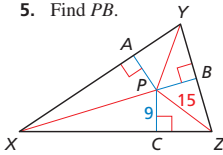


4. Find GA .

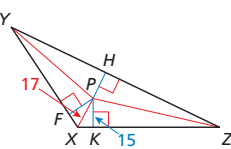


In Exercises 5 and 6, the angle bisectors of $\triangle XYZ$ intersect at point P and are shown in red. Find the indicated measure.

5. Find PB .



6. Find HP .

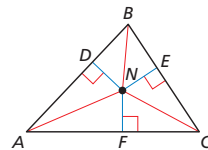


In Exercises 7–10, find the coordinates of the circumcenter of the triangle with the given vertices. (See Example 2.)

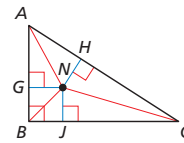
- $A(2, 6), B(8, 6), C(8, 10)$
- $D(-7, -1), E(-1, -1), F(-7, -9)$
- $H(-10, 7), J(-6, 3), K(-2, 3)$
- $L(3, -6), M(5, -3), N(8, -6)$

In Exercises 11–14, N is the incenter of $\triangle ABC$. Use the given information to find the indicated measure. (See Example 3.)

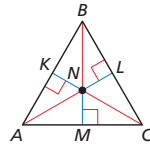
11. $ND = 6x - 2$
 $NE = 3x + 7$
Find NF .



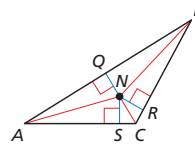
12. $NG = x + 3$
 $NH = 2x - 3$
Find NJ .



13. $NK = 2x - 2$
 $NL = -x + 10$
Find NM .

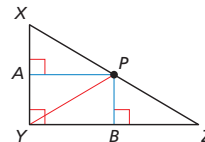


14. $NQ = 2x$
 $NR = 3x - 2$
Find NS .



15. **PROBLEM SOLVING** P is the circumcenter of $\triangle XYZ$. Use the given information to find PZ .

$PX = 3x + 2$
 $PY = 4x - 8$



Assignment Guide and Homework Check

ASSIGNMENT

- Basic:** 1, 2, 3–27 odd, 28, 45, 46, 52–59
Average: 1, 2–28 even, 41–45 odd, 46, 52–59
Advanced: 1, 2, 4–8 even, 14–36 even, 43–59
- HOMEWORK CHECK**
- Basic:** 3, 5, 7, 13, 28
Average: 4, 6, 8, 14, 18
Advanced: 6, 14, 18, 34, 43

ANSWERS

- concurrent
- the fourth triangle; This triangle shows the incenter of the triangle, but the other three show the circumcenter.
- 9
- 11
- 9
- 15
- (5, 8)
- (-4, -5)
- (-4, 9)
- (5.5, -5.5)
- 16
- 9
- 6
- 4
- 32

Dynamic Teaching Tools

Dynamic Assessment & Progress Monitoring Tool

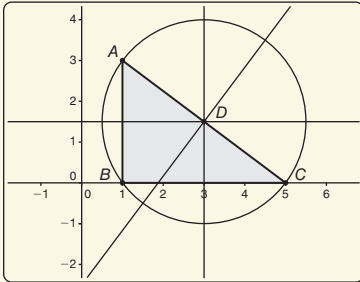
Interactive Whiteboard Lesson Library

Dynamic Classroom with Dynamic Investigations

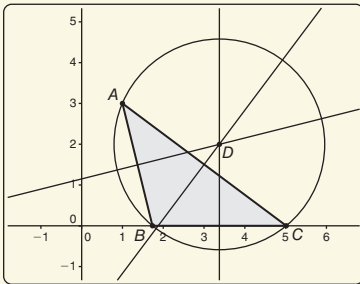
ANSWERS

16. 31

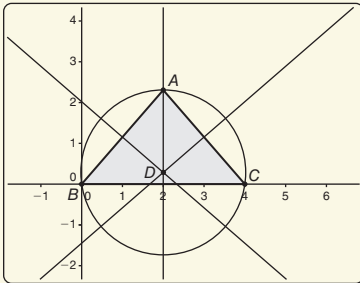
17. *Sample answer:*



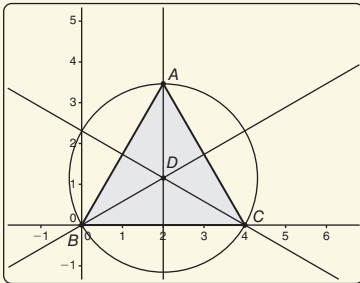
18. *Sample answer:*



19. *Sample answer:*



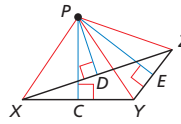
20. *Sample answer:*



16. **PROBLEM SOLVING** P is the circumcenter of $\triangle XYZ$. Use the given information to find PY .

$$PX = 4x + 3$$

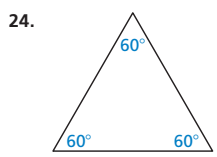
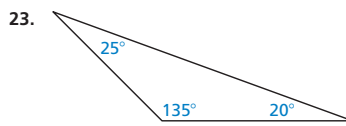
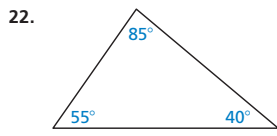
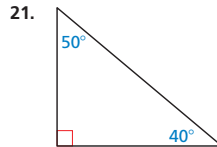
$$PZ = 6x - 11$$



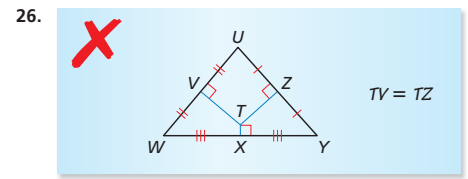
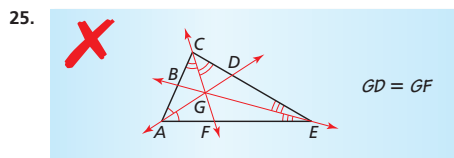
CONSTRUCTION In Exercises 17–20, draw a triangle of the given type. Find the circumcenter. Then construct the circumscribed circle.

17. right 18. obtuse
19. acute isosceles 20. equilateral

CONSTRUCTION In Exercises 21–24, copy the triangle with the given angle measures. Find the incenter. Then construct the inscribed circle.



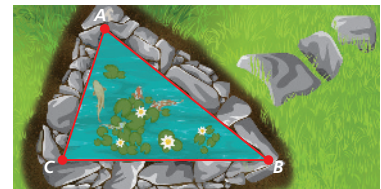
ERROR ANALYSIS In Exercises 25 and 26, describe and correct the error in identifying equal distances inside the triangle.



27. **MODELING WITH MATHEMATICS** You and two friends plan to meet to walk your dogs together. You want the meeting place to be the same distance from each person's house. Explain how you can use the diagram to locate the meeting place. (*See Example 1.*)



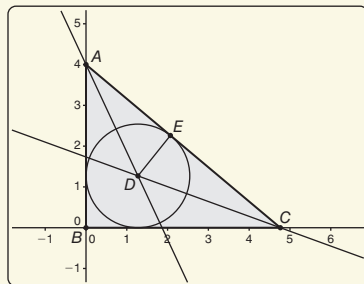
28. **MODELING WITH MATHEMATICS** You are placing a fountain in a triangular koi pond. You want the fountain to be the same distance from each edge of the pond. Where should you place the fountain? Explain your reasoning. Use a sketch to support your answer. (*See Example 4.*)



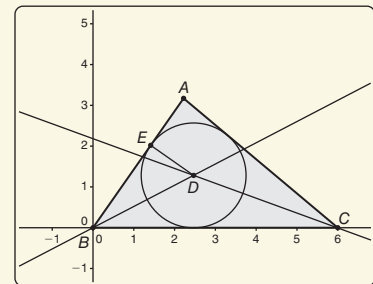
CRITICAL THINKING In Exercises 29–32, complete the statement with *always*, *sometimes*, or *never*. Explain your reasoning.

29. The circumcenter of a scalene triangle is _____ inside the triangle.
30. If the perpendicular bisector of one side of a triangle intersects the opposite vertex, then the triangle is _____ isosceles.
31. The perpendicular bisectors of a triangle intersect at a point that is _____ equidistant from the midpoints of the sides of the triangle.
32. The angle bisectors of a triangle intersect at a point that is _____ equidistant from the sides of the triangle.

21. *Sample answer:*



22. *Sample answer:*



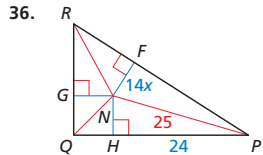
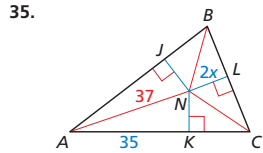
23–32. See Additional Answers.

CRITICAL THINKING In Exercises 33 and 34, find the coordinates of the circumcenter of the triangle with the given vertices.

33. $A(2, 5), B(6, 6), C(12, 3)$

34. $D(-9, -5), E(-5, -9), F(-2, -2)$

MATHEMATICAL CONNECTIONS In Exercises 35 and 36, find the value of x that makes N the incenter of the triangle.

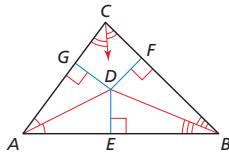


37. **PROOF** Where is the circumcenter located in any right triangle? Write a coordinate proof of this result.

38. **PROVING A THEOREM** Write a proof of the Incenter Theorem (Theorem 6.6).

Given $\triangle ABC$, \overline{AD} bisects $\angle CAB$,
 \overline{BD} bisects $\angle CBA$, $\overline{DE} \perp \overline{AB}$, $\overline{DF} \perp \overline{BC}$,
and $\overline{DG} \perp \overline{CA}$

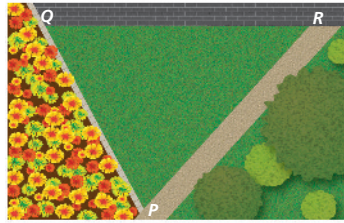
Prove The angle bisectors intersect at D , which is equidistant from \overline{AB} , \overline{BC} , and \overline{CA} .



39. **WRITING** Explain the difference between the circumcenter and the incenter of a triangle.

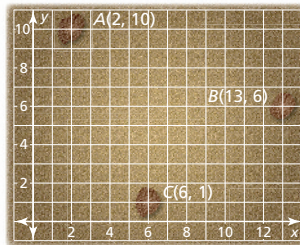
40. **REASONING** Is the incenter of a triangle ever located outside the triangle? Explain your reasoning.

41. **MODELING WITH MATHEMATICS** You are installing a circular pool in the triangular courtyard shown. You want to have the largest pool possible on the site without extending into the walkway.



- Copy the triangle and show how to install the pool so that it just touches each edge. Then explain how you can be sure that you could not fit a larger pool on the site.
- You want to have the largest pool possible while leaving at least 1 foot of space around the pool. Would the center of the pool be in the same position as in part (a)? Justify your answer.

42. **MODELING WITH MATHEMATICS** Archaeologists find three stones. They believe that the stones were once part of a circle of stones with a community fire pit at its center. They mark the locations of stones A, B, and C on a graph, where distances are measured in feet.



- Explain how archaeologists can use a sketch to estimate the center of the circle of stones.
- Copy the diagram and find the approximate coordinates of the point at which the archaeologists should look for the fire pit.

43. **REASONING** Point P is inside $\triangle ABC$ and is equidistant from points A and B . On which of the following segments must P be located?

- \overline{AB}
- the perpendicular bisector of \overline{AB}
- \overline{AC}
- the perpendicular bisector of \overline{AC}

Section 6.2 Bisectors of Triangles 321

ANSWERS

33. $(\frac{35}{6}, -\frac{11}{6})$

34. $(-4.9, -4.9)$

35. $x = 6$

36. $x = \frac{1}{2}$

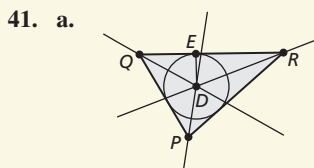
37. The circumcenter of any right triangle is located at the midpoint of the hypotenuse of the triangle.

Let $A(0, 2b)$, $B(0, 0)$, and $C(2a, 0)$ represent the vertices of a right triangle where $\angle B$ is the right angle. The midpoint of \overline{AB} is $M_{\overline{AB}}(0, b)$. The midpoint of \overline{BC} is $M_{\overline{BC}}(a, 0)$. The midpoint of \overline{AC} is $M_{\overline{AC}}(a, b)$. Because \overline{AB} is vertical, its perpendicular bisector is horizontal. So, the equation of the horizontal line passing through $M_{\overline{AB}}(0, b)$ is $y = b$. Because \overline{BC} is horizontal, its perpendicular bisector is vertical. So, the equation of the vertical line passing through $M_{\overline{BC}}(a, 0)$ is $x = a$. The circumcenter of $\triangle ABC$ is the intersection of perpendicular bisectors, $y = b$ and $x = a$, which is (a, b) . This point is also the midpoint of \overline{AC} .

38. Because $\overline{DE} \perp \overline{AB}$, $\overline{DF} \perp \overline{BC}$, and $\overline{DG} \perp \overline{CA}$, $\angle DFB$, $\angle DEB$, $\angle DEA$, and $\angle DGA$ are congruent right angles. Also, by definition of angle bisector, $\angle DBF \cong \angle DBE$ and $\angle DAE \cong \angle DAG$. In addition, $\overline{DB} \cong \overline{DB}$ and $\overline{DA} \cong \overline{DA}$ by the Reflexive Property of Congruence (Thm. 2.1). So, $\triangle DFB \cong \triangle DEB$ and $\triangle DEA \cong \triangle DGA$ by the AAS Congruence Theorem (Thm. 5.11). Next, because corresponding parts of congruent triangles are congruent, $\overline{DF} \cong \overline{DE}$ and $\overline{DG} \cong \overline{DE}$. By the Transitive Property of Congruence (Thm. 2.1), $\overline{DF} \cong \overline{DE} \cong \overline{DG}$. So, point D is equidistant from \overline{AB} , \overline{BC} , and \overline{CA} . Because D is equidistant from \overline{CA} and \overline{CB} , by the Converse of the Angle Bisector Theorem (Thm. 6.4), point D is on the angle bisector of $\angle ACB$. So, the angle bisectors intersect at point D .

39. The circumcenter is the point of intersection of the perpendicular bisectors of the sides of a triangle, and it is equidistant from the vertices of the triangle. In contrast, the incenter is the point of intersection of the angle bisectors of a triangle, and it is equidistant from the sides of the triangle.

40. no; Because the incenter is the center of an inscribed circle, it must be inside the triangle.



Because this circle is inscribed in the triangle, it is the largest circle that fits inside the triangle without extending into the boundaries.

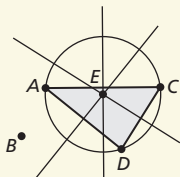
b. yes; You would keep the center of the pool as the incenter of the triangle, but you would make the radius of the pool at least 1 foot shorter.

42. See Additional Answers.

43. B

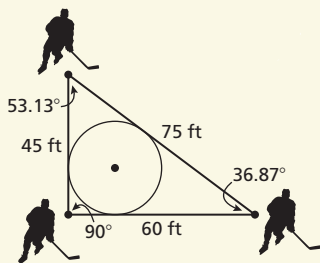
ANSWERS

44. no; When you find the circumcenter of three of the points and draw the circle that circumscribes those three points, it does not pass through the fourth point. An example of one circle is shown.



45. yes; In an equilateral triangle, each perpendicular bisector passes through the opposite vertex and divides the triangle into two congruent triangles. So, it is also an angle bisector.
46. incenter
47. a. equilateral; 3; In an equilateral triangle, each perpendicular bisector also bisects the opposite angle.
- b. scalene; 6; In a scalene triangle, none of the perpendicular bisectors will also bisect an angle.

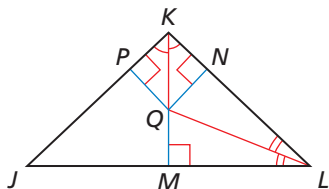
48. *Sample answer:*



49. angle bisectors; about 2.83 in.
50. perpendicular bisectors; (10, 10); 10
- 51–59. See Additional Answers.

Mini-Assessment

- Find the coordinates of the circumcenter of $\triangle ABC$ with vertices $A(0, 5)$, $B(4, 5)$, and $C(4, -1)$. (2, 2)
- In the figure shown, $QP = 2x + 9$ and $QM = 5x - 3$. Find QN .



$QN = 17$

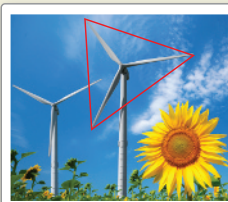
44. **CRITICAL THINKING** A high school is being built for the four towns shown on the map. Each town agrees that the school should be an equal distance from each of the four towns. Is there a single point where they could agree to build the school? If so, find it. If not, explain why not. Justify your answer with a diagram.



45. **MAKING AN ARGUMENT** Your friend says that the circumcenter of an equilateral triangle is also the incenter of the triangle. Is your friend correct? Explain your reasoning.

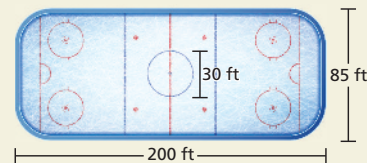
46. **HOW DO YOU SEE IT?**

The arms of the windmill are the angle bisectors of the red triangle. What point of concurrency is the point that connects the three arms?



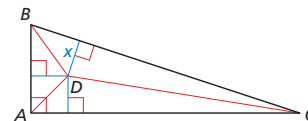
47. **ABSTRACT REASONING** You are asked to draw a triangle and all its perpendicular bisectors and angle bisectors.
- For which type of triangle would you need the fewest segments? What is the minimum number of segments you would need? Explain.
 - For which type of triangle would you need the most segments? What is the maximum number of segments you would need? Explain.

48. **THOUGHT PROVOKING** The diagram shows an official hockey rink used by the National Hockey League. Create a triangle using hockey players as vertices in which the center circle is inscribed in the triangle. The center dot should be the incenter of your triangle. Sketch a drawing of the locations of your hockey players. Then label the actual lengths of the sides and the angle measures in your triangle.



COMPARING METHODS In Exercises 49 and 50, state whether you would use *perpendicular bisectors* or *angle bisectors*. Then solve the problem.

49. You need to cut the largest circle possible from an isosceles triangle made of paper whose sides are 8 inches, 12 inches, and 12 inches. Find the radius of the circle.
50. On a map of a camp, you need to create a circular walking path that connects the pool at (10, 20), the nature center at (16, 2), and the tennis court at (2, 4). Find the coordinates of the center of the circle and the radius of the circle.
51. **CRITICAL THINKING** Point D is the incenter of $\triangle ABC$. Write an expression for the length x in terms of the three side lengths AB , AC , and BC .



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

The endpoints of \overline{AB} are given. Find the coordinates of the midpoint M . Then find AB . (Section 1.2 and Section 1.3)

52. $A(-3, 5)$, $B(3, 5)$

53. $A(2, -1)$, $B(10, 7)$

54. $A(-5, 1)$, $B(4, -5)$

55. $A(-7, 5)$, $B(5, 9)$

Write an equation of the line passing through point P that is perpendicular to the given line. Graph the equations of the lines to check that they are perpendicular. (Section 3.6)

56. $P(2, 8)$, $y = 2x + 1$

57. $P(6, -3)$, $y = -5$

58. $P(-8, -6)$, $2x + 3y = 18$

59. $P(-4, 1)$, $y + 3 = -4(x + 3)$

If students need help...

Resources by Chapter

- Practice A and Practice B
- Puzzle Time

Student Journal

- Practice

Differentiating the Lesson
Skills Review Handbook

If students got it...

Resources by Chapter

- Enrichment and Extension
- Cumulative Review

Start the *next* Section